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Large-Scale Price Optimization via Network Flow

Shinji Ito, Ryohei Fujimaki

Our goal: profit maximization by optimizing prices

What is the best pricing strategy?

Strategy 1		Strategy 2
: $ 1.3	Price	: $ 1.0
	Sales quantity	
?	Profit	?

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What is the best pricing strategy?

Strategy 1		Strategy 2
: $ 1.3	Price	: $ 1.0
×		×
	Sales quantity	
=		=
$ 5.2	Profit	$ 6.0

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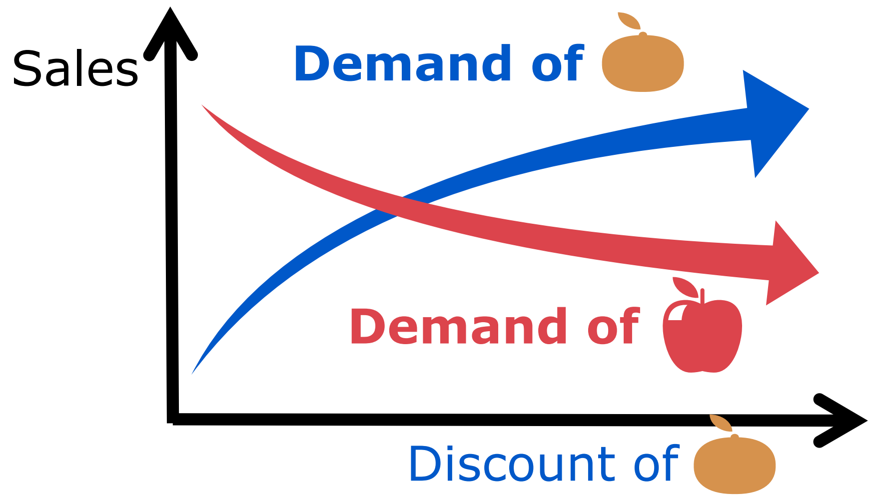
Complicated structure in price optimization

Changing the price of one product affects other's sales

- **Cannibalization**:

  Growing the sales of

  makes the sales of down

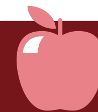
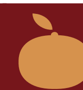


	Price	Quantity	Profit
Product 1	**$1.3 ➔ $1.0**	**+200**	**+ $80**
Product 2	**$1.2**	**-100**	**- $120**
	Gross profit	**- $40**	

Predictive price optimization and its difficulty

- Recent advanced ML reveals relationship between prices and sales quantities

Input:
pos data

	 Price	 Price	 Sales	 Sales	...
Day 1	**$1.3**	**$1.0**	**2**	**2**	...
Day 2	**$1.2**	**$1.0**	**4**	**2**	...
⋮	⋮	⋮	⋮	⋮	

Machine learning

Predictive model: $\text{sales} = f(\text{prices})$

Predictive price optimization and its difficulty

- Recent advanced ML reveals relationship between prices and sales quantities

Input:
pos data

	(orange) Price	(apple) Price	(orange) Sales	(apple) Sales	...
Day 1	**$1.3**	**$1.0**	**2**	**2**	...
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⋮	⋮	⋮	⋮	⋮	

Machine learning

Predictive model: $\text{sales} = f(\text{prices})$

Optimization (NP-hard) **[This work]**

Output:
optimal prices

Our contribution

Scalable algorithm for price optimization

Based on:

1. Submodularity behind pricing
2. Network flow algorithm
3. Supermodular relaxation

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Achieved:

- Can deal with thousands of products
- High accuracy for real-data problem

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Objective function of price optimization

Want to maximize is the gross profit ℓ

Gross profit:

$$\ell(p_1, p_2, \dots, p_M) = \sum_{i=1}^{M} (p_i - c_i) q_i$$

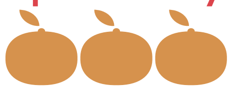
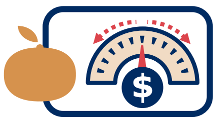
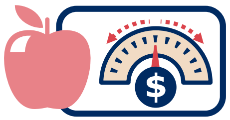
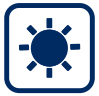
- i: product id
- p_i: price
- c_i: cost
- q_i: sales quantity — Unknown, but predictable

Predictive model for sales quantity

Sales quantity q_i is a function in prices p_i

$$\underline{q_1(p,r)} = f_{11}(\underline{p_1}) + f_{12}(\underline{p_2}) + \cdots \quad + g_{11}(\underline{r_1}) + g_{12}(\underline{r_2}) + \cdots$$

sales quantity — price — price — weather — calendar


Use historical data to infer f_{ij}, g_{ij}

Ex: $f_{ij}(p_j) = a_{ij}p_j^2 + b_{ij}p_j + c_{ij}$ (polynomial model)

$f_{ij}(p_j) = \exp(\alpha_{ij}p_j + \beta_{ij})$, (generalized linear model)

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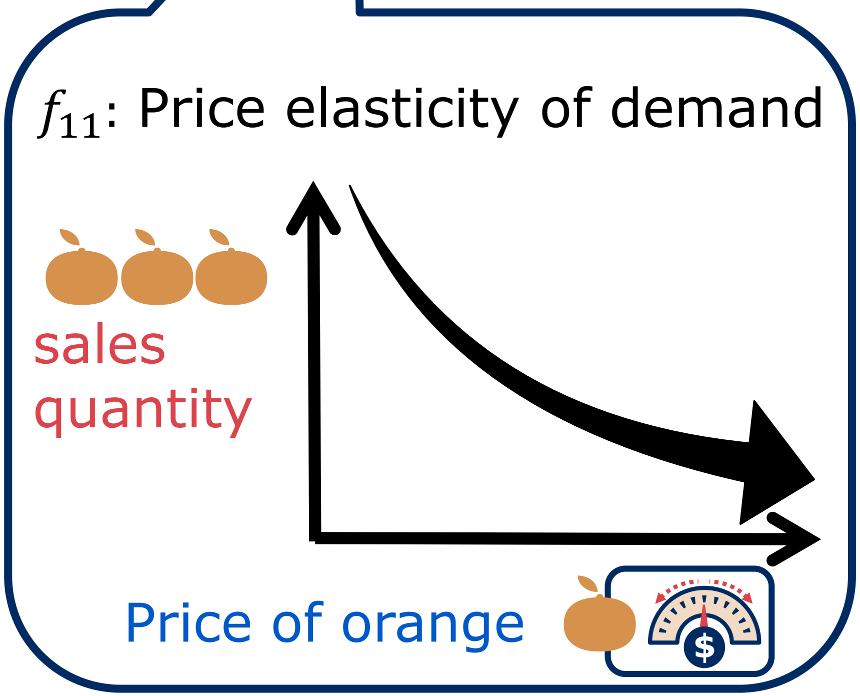
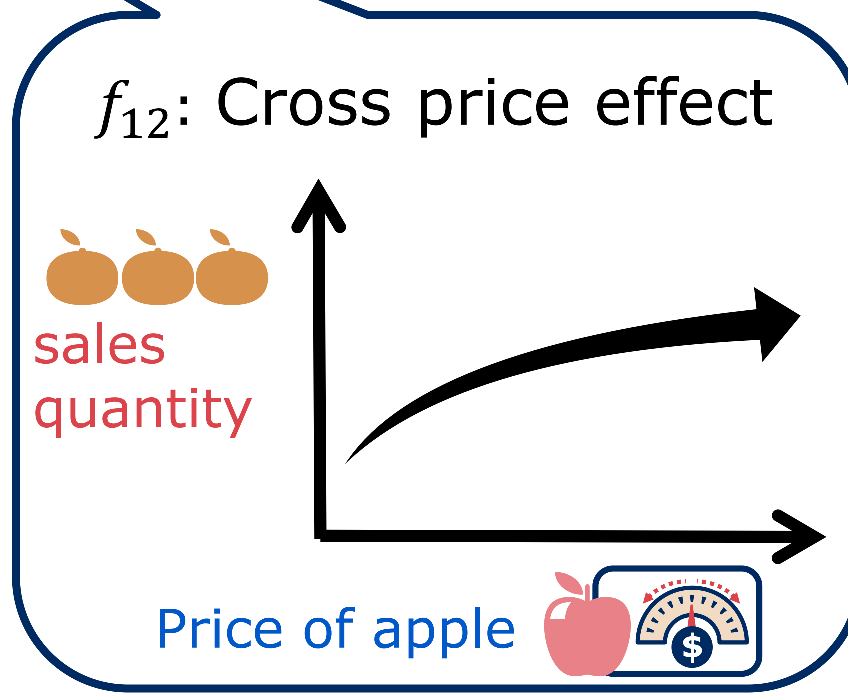
sales quantity | price | price | Weather | calendar





Substitute goods in price optimization

Many substitute goods in price optimization

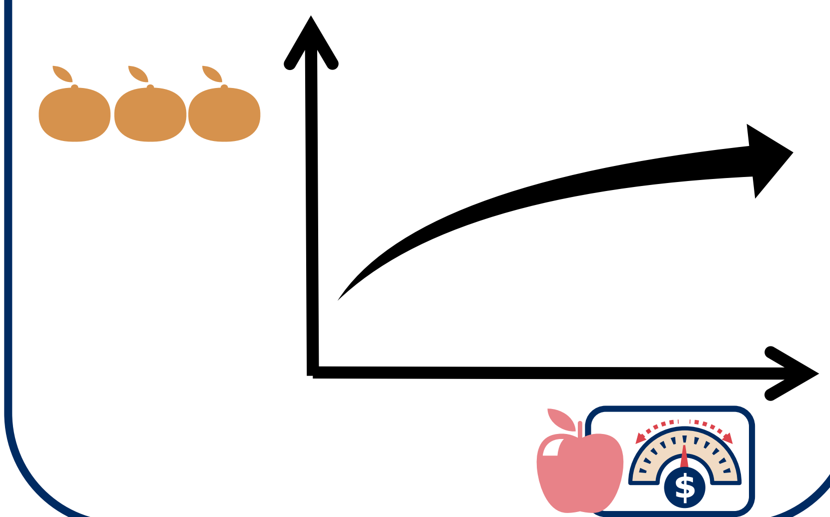
$$\underline{q_1(p,r)} = f_{11}(\underline{p_1}) + f_{12}(\underline{p_2}) + \cdots \quad + g_{11}(\underline{r_1}) + g_{12}(\underline{r_2}) + \cdots$$

and : *Substitute goods*

⇕ def

Discounting apple makes the sales of orange down (cannibalization)

f_{12}: Cross price effect



Optimization is NP-hard

Gross profit

Maximize $\ell(p) = \sum_{i=1}^{M} (p_i - c_i) q_i(p)$

list price, 5%OFF … 20%OFF

Subject to $p_i \in \{P_{i1}, P_{i2}, \ldots, P_{ik}\}$

Discrete price candidates

A commercial solver takes >24[h] for 50 products

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1. Introduction
2. Problem definition
3. **Scalable price optimization algorithm**
4. Experiments

Our contribution

Scalable algorithm for price optimization

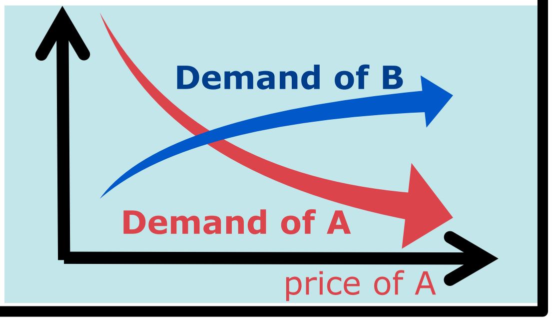
Based on:

1. Submodularity behind pricing
2. Network flow algorithm
3. Supermodular relaxation

Connection between substitute goods and submodular

Substitute goods:

Discounting 🍎 ⇒ less sales of 🍊



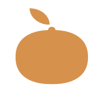
Theorem [Substitute goods ⇒ supermodular]

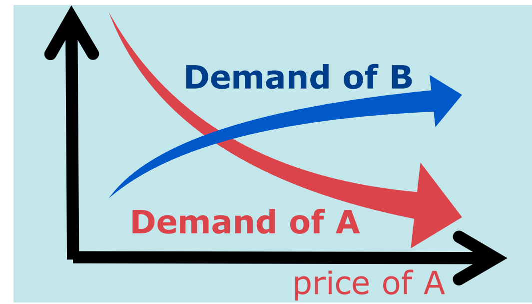
If all pairs of products are substitute goods or independent

⇒ $\ell(p)$ is a supermodular function

Connection between substitute goods and submodular

Substitute goods:

Discounting  ⇒ less sales of 



Theorem [Substitute goods ⇒ supermodular]

If all pairs of products are substitute goods or independent

⇒ $\ell(p)$ is a supermodular function

maximized in polynomial time

[Iwata, Fleischer, Fujishige (2001)]

Two issues in Key idea 1

- If all products are substitute goods or independent, profit can be maximized in polynomial time

Two issues in Key idea 1

- If all products are substitute goods or independent, profit can be maximized in polynomial time

- This approach is still impractical because of two issues

Issue 1: General supermodular maximization is slow

$\sim O(n^5)$

Issue 2: Substitute goods assumption is too restrictive

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⇒ Resolved by 2. network flow: $O(n^2) \sim O(n^3)$

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⇒ Resolved by 3. supermodular relaxation

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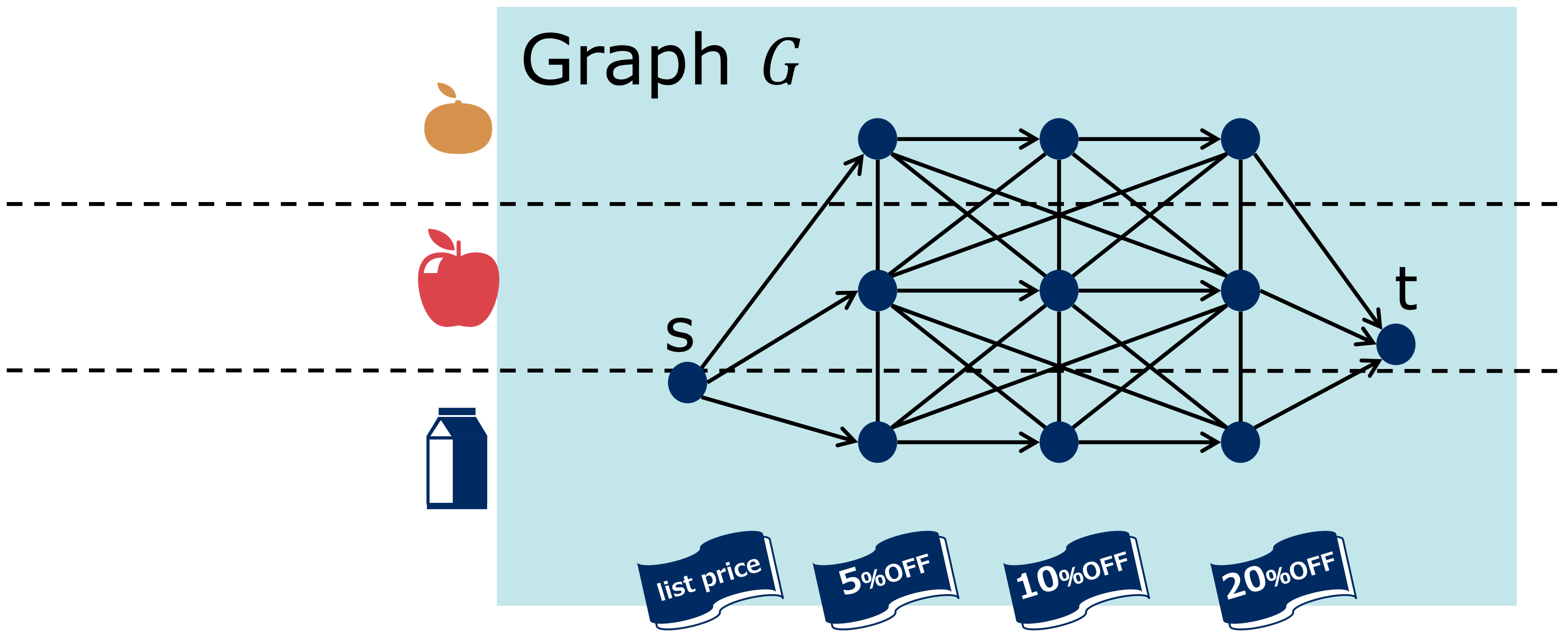
Applicable for non-substitute

Substitute goods price optimization ⇔ Minimum Cut

- Maximizing $\ell(p)$ ⇔ Finding minimum s-t cut

  :solved efficiently by network flow

  [Ford, Fulkerson (1956)], [Orlin (2013)]



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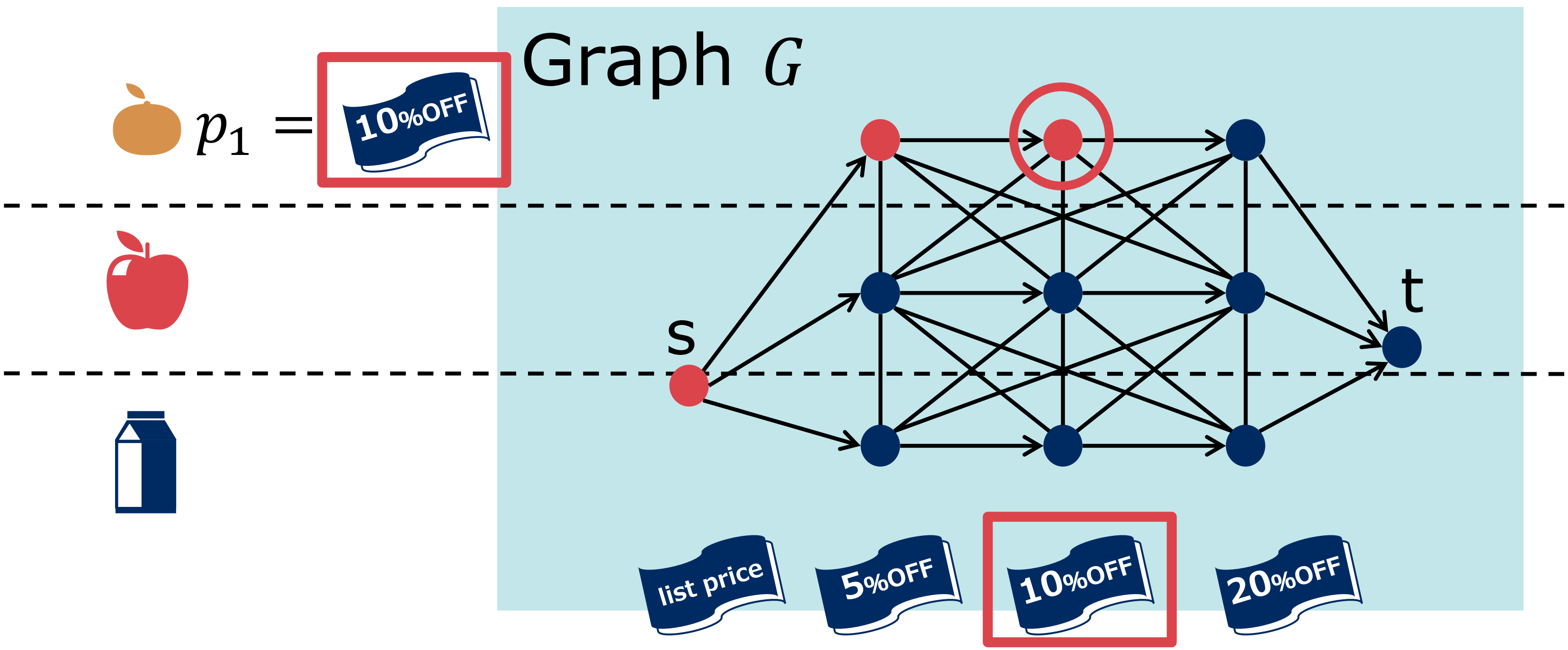

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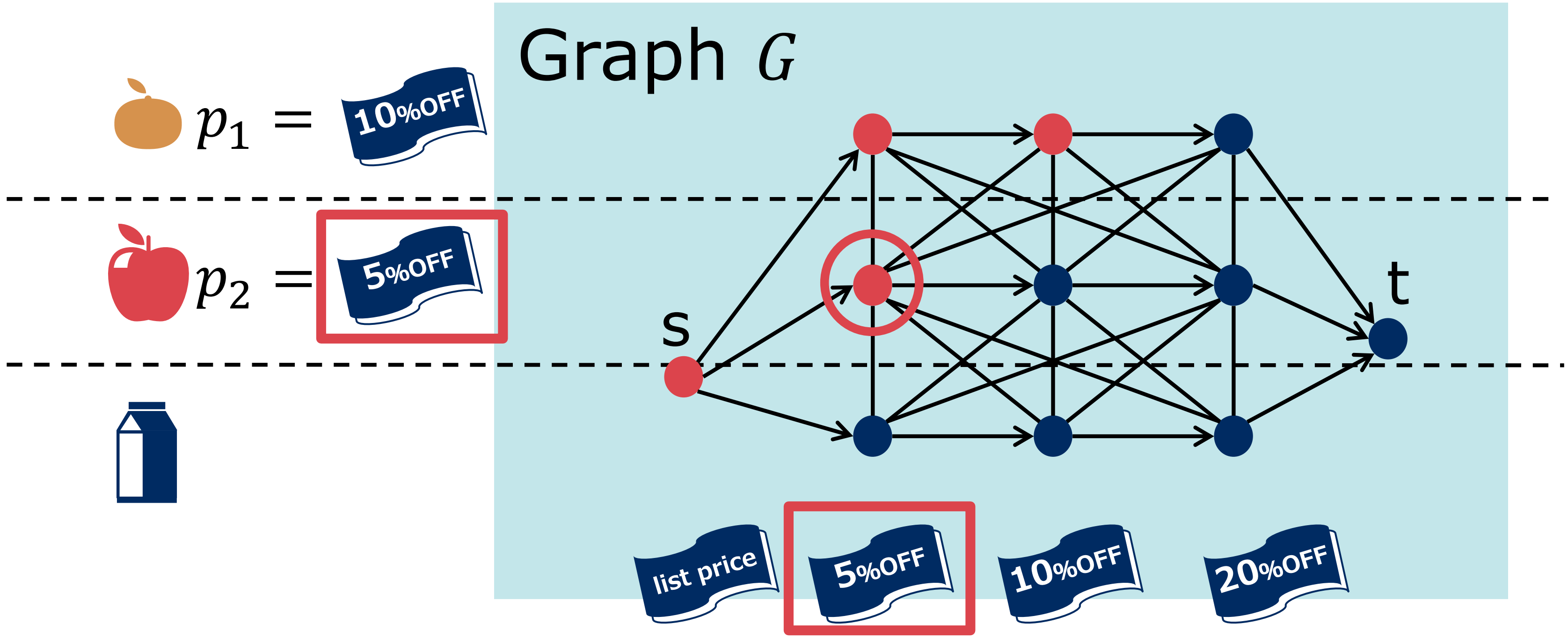

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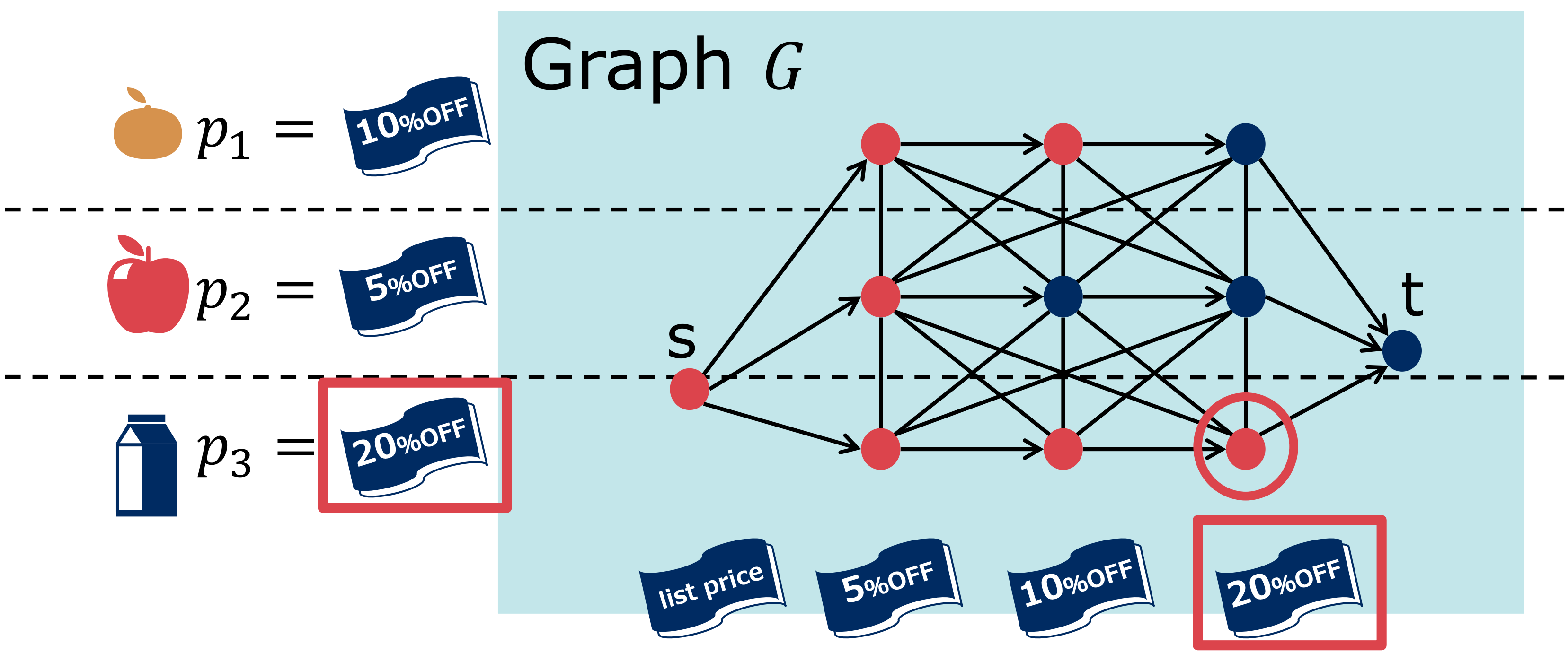

Substitute goods price optimization ⇔ Minimum Cut

Maximizing $\ell(p)$ ⇔ Finding minimum s-t cut

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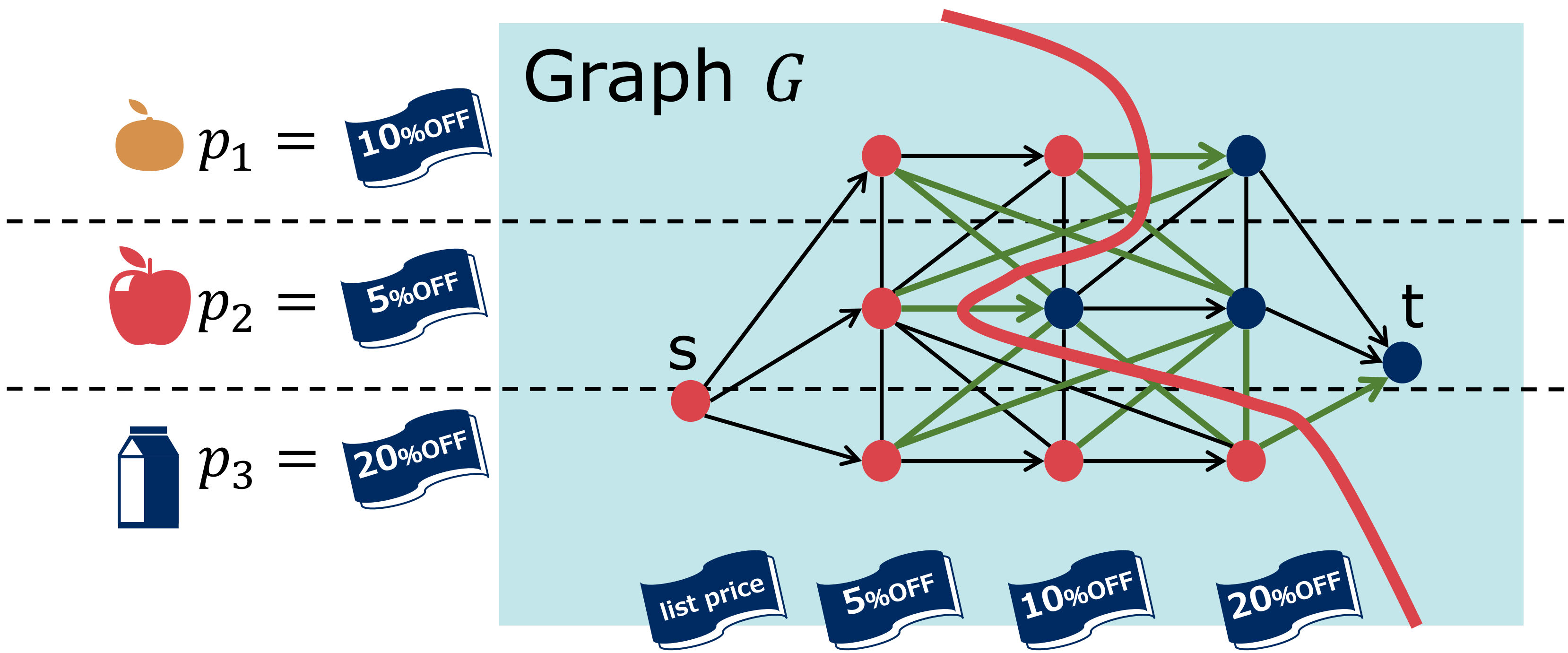
$$\ell(p) = \text{constant} - (\text{capacity of st-cut of graph } G)$$



Two issues in Key idea 1

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$O(n^5)$

➡ Resolved by 2. network flow:

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➡ Resolved by 3. supermodular relaxation

Applicable for non-substitute

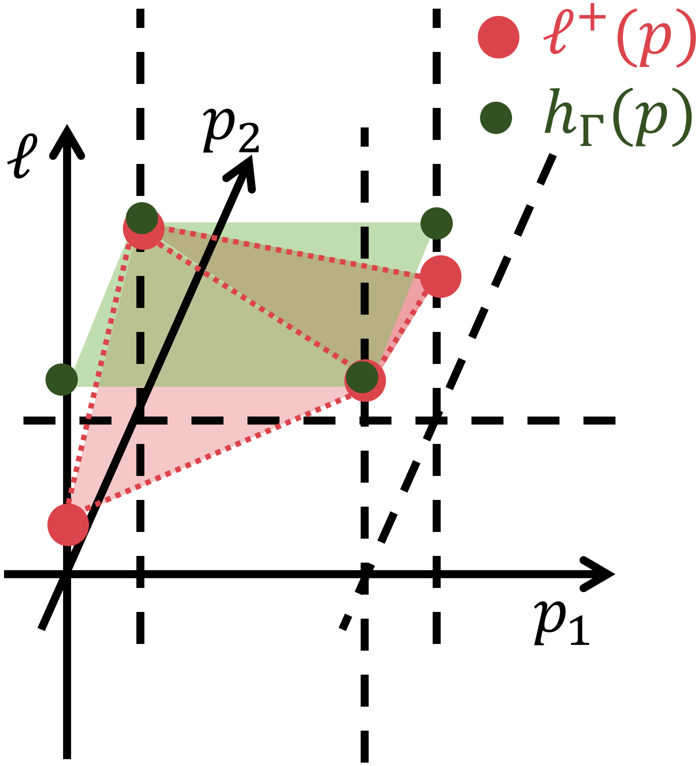
Non-SGP case: supermodular relaxation

- ∃ non-substitute goods
  ⇒ approximate ℓ by supemodular function

$$\ell(p) = \underbrace{\ell^-(p)}_{\text{supermodular}} + \underbrace{\ell^+(p)}_{\text{submodular}}$$

$$\leq \underbrace{\boxed{\ell^-(p) + \underbrace{h_\Gamma(p)}_{\text{modular}}}}_{\text{supermodular}}$$

⇒ maximized via network flow

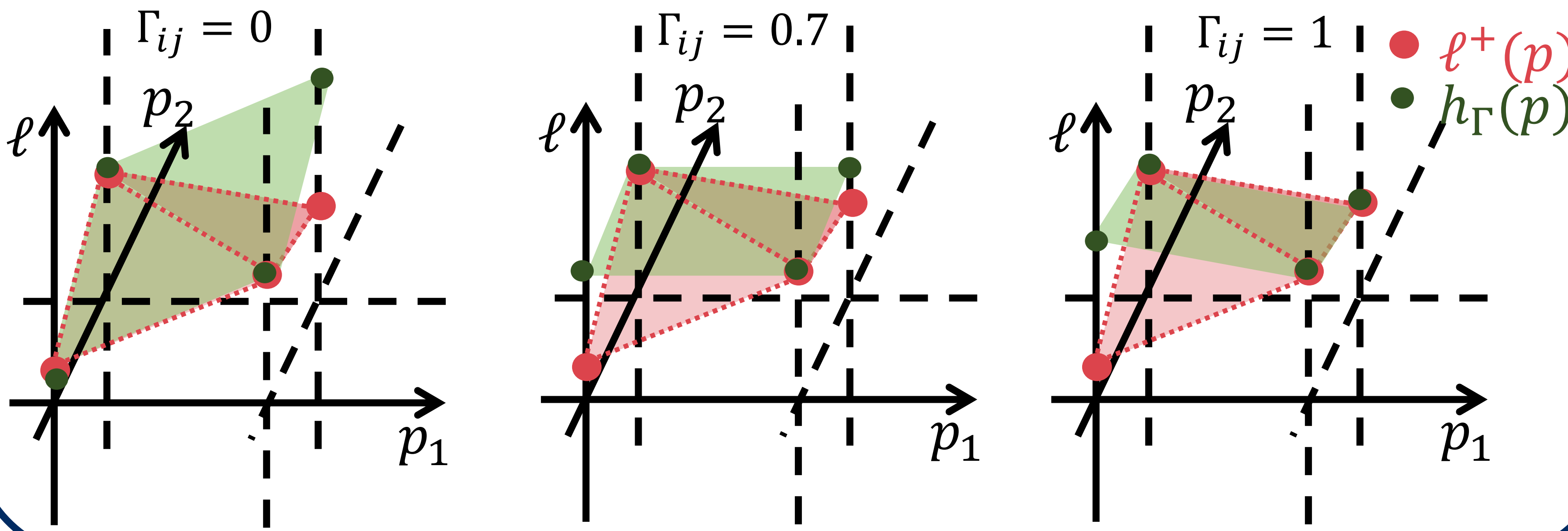


Non-SGP case: supermodular relaxation

- Many possibilities of relaxation function

  Better relaxation is chosen automatically

Relaxation changes depending on $\Gamma \in [0,1]^{n \times n}$



Proposed approximation algorithm: fast and give solution with only <1% loss

Existing methods

	Past data	Proposed	QPBO	Others
Computing Time		**36 [s]**	**964 [s]**	**> 1day**
Achieved Profit	**1.40 M**	**1.88 M**	**1.25 M**	**Nan**
Upper bound		**1.90 M**	**1.89 M**	**Nan**

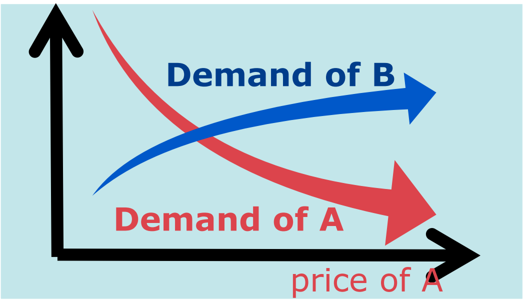
- Real-world retail data* of a supermarket

*provided by KSP-SP Co., LTD.

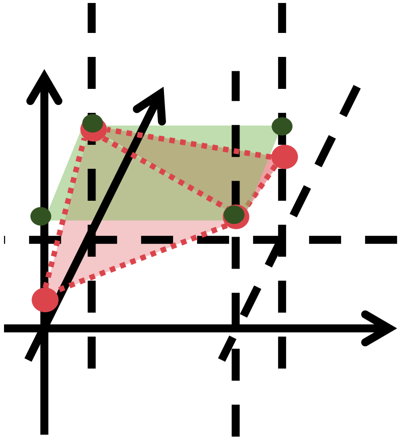
- 1000 products
- Compared with SDP, QPBO, QPBOI [Rother et.al (2007)]

Conclusion

- Constructed a scalable price optimization algorithm by
  - Associating substitute goods with supermodularity



  - Using network flow and supermodular relaxation



- Future work: how to cope with the errors in objective?

  e.g., by robust optimization?

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