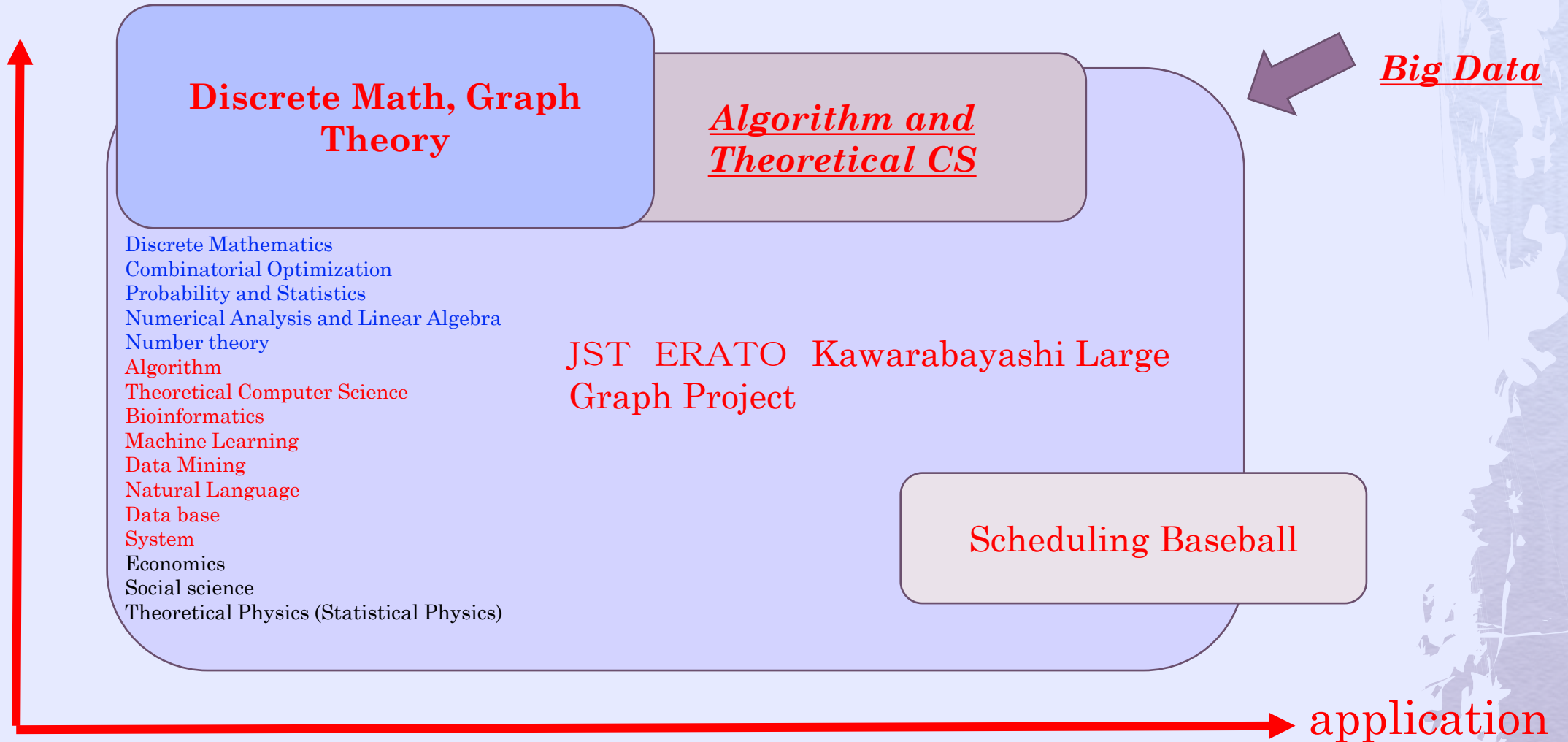


My Research

Depth



ピックス

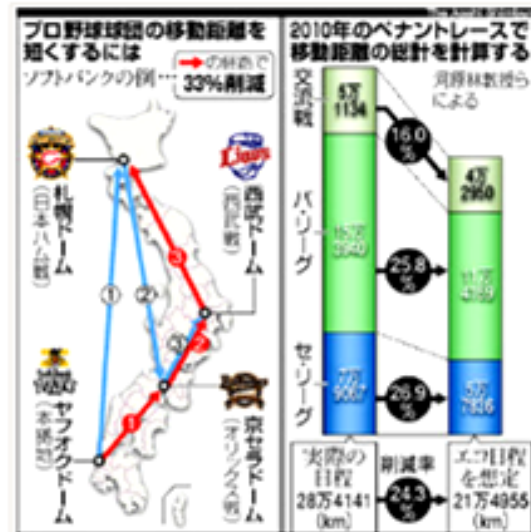
選抜大会32校決定 東京都知事選 田中、ヤンキースへ ソチ五輪

スポーツ > 野球 > プロ野球 > 記事

2013年3月18日 10時10分

ツイート 162 おすすめ 234 8+1 44 81 30 印刷 メール 印刷

プロ野球日程を瞬時に計算 数学者がソフトを開発

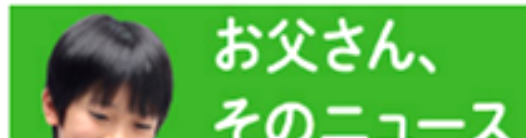


【藤島真人】1カ月以上かかるプロ野球の日程づくりが、あっという間にできる ソフトウェア を数学者が開発した。日程を工夫するだけで、球団の移動距離を2割以上減らすこともできるという。シーズン開幕控え、彼らの計算式と現実とのギャップはいかに。

開発したのは、国立情報学研究所の河原林健一教授と星野リチャード・前外来研究員。2人は効率の良い鉄道網や電気回路の設計などにも応用できる「グラフ理論」と呼ばれる分野の研究者だ。

ソフト開発は、3年前の3月、カナダからリチャードさんが来日し、たまたま千葉ロッテの本拠地の近くに住んだことがきっかけ。鉄道網のような「グラフ」を球団の移動に置き換え、最適な日程をはじき出せないかを考えた。

【PR】



Graph Theory and Sports Scheduling

Richard Hoshino and Ken-ichi Kawarabayashi

Introduction

The effects of global warming have been well documented, especially in recent years. As a result, the majority of countries have made a commitment to reducing their greenhouse gas emissions, including many whose national governments have made ambitious and unrealistic promises. Meeting these targets will require a coordinated effort from policymakers, businesses, and large industries, and numerous creative solutions will need to be implemented to achieve the desired goal. One potential solution is based on discrete mathematics, where combinatorial and graph-theoretic techniques are applied to scheduling optimization, leading to economic and environmental benefits.

There are many practical roles for mathematically optimal schedules that reduce total travel distance, including supply-chain logistics and airplane flight assignments. In this paper we describe how to optimize the regular-season schedule for

converted into a much simpler shortest-path problem. As we describe in this paper, the answer to this question is affirmative. Consequently, we have succeeded in generating the distance-optimal NPB regular-season schedule which retains all of the league's constraints that ensure competitive balance while reducing the total travel distance by 24.3%, or nearly 70,000 kilometers, as compared to the 2010 season schedule.

To solve the NPB scheduling problem, we have generalized and extended the Traveling Tournament Problem (TTP), a well-known topic in sports scheduling [10]. Our research has produced five papers, [4], [5], [6], [7], [8], describing the theoretical aspects of the problem, providing various heuristics for generating distance-optimal intra-league and inter-league schedules, and applying the results to optimize the NPB league schedule.

Shortly after introducing the Traveling Tournament Problem [2], Easton et al. formed a consulting

company to develop schedules for professional sports leagues. Their company, the Sports Scheduling Group, has received the contract to produce the regular-season schedule for Major League Baseball in six of the past seven years. Having now completed all of our research on NPB scheduling, our hope is to obtain the contract to produce future NPB regular-season schedules. We are excited by the possibility of sharing our expertise and passion with Nippon Professional Baseball, working in partnership with the league to produce schedules that save money and reduce greenhouse gas emissions, thus making an important contribution to Japan, both economically and environmentally.

There are many practical roles for mathematically optimal schedules that reduce total travel distance, including supply-chain logistics and airplane flight assignments. In this paper we describe how to optimize the regular-season schedule for Nippon Professional Baseball (NPB), Japan's most popular professional sports league, with annual revenues topping one billion U.S. dollars.

In the authors' background as graph theorists, this research was motivated by the innocent question of whether NPB scheduling could be

converted into a much simpler shortest-path problem.

As we describe in this paper, the answer to this question is affirmative. Consequently, we have succeeded in generating the distance-optimal NPB regular-season schedule which retains all of the league's constraints that ensure competitive balance while reducing the total travel distance by 24.3%, or nearly 70,000 kilometers, as compared to the 2010 season schedule.

To solve the NPB scheduling problem, we have generalized and extended the Traveling Tournament Problem (TTP), a well-known topic in sports scheduling [10]. Our research has produced five papers, [4], [5], [6], [7], [8], describing the theoretical aspects of the problem, providing various heuristics for generating distance-optimal intra-league and inter-league schedules, and applying the results to optimize the NPB league schedule.

Hoshino is a mathematics tutor at Quest University. His email address is richard.hoshino@questu.ca.

Kawarabayashi is a professor at the National Institute of Informatics, Japan. His email address is k_keniti@nii.ac.jp.

<https://dx.doi.org/10.1090/noti1010>

STOCとは？

- ◆ STOC, FOCS, SODA: 理論計算機科学のトップ会議
- ◆ また暗号(Crypto), プライバシー(Privacy), 量子計算量でもベストな結果が発表される！
- ◆ しかし実際は, , , 「難関」数学！
- ◆ 誰も実装はしない。実装にはほぼ意味がない。おそらく興味すらない。
→ 理論家の論文をそのまま実装してはだめ
- ◆ 近年のトレンド: 離散数学, 解析, 確率, 整数論, 幾何, 代数などから, 超「難関」数学の応用と拡張が中心
- ◆ 数学のビッグネームの参入: Terry Tao, Tim Gowers, Laszlo Lovasz, Lex Schrijver, Noga Alon, Endre Szemerédi, Assaf Naor, Yuval Peres, Jeff Cheeger, Bruce Kelner… (フィールズ賞, アーベル賞, あるいはアーベル賞候補者)

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[Baruch Awerbuch](#) (59)
[Robert Endre Tarjan](#) (59)
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[Ken-ichi Kawarabayashi](#) (46)
[Zvi Galil](#) (46)
[Ran Raz](#) (45)

理論の世界： STOC, FOCS, SODA世界

チューリング賞受賞者や、ネバンリンナ賞受賞が、
大量にいる世界 + 未来のチューリング賞受賞候補が
たくさんいる世界(おそらくチューリング賞を最も輩出している分野)
過去30年のチューリング賞受賞者

Richard Karp (1985), Hopcroft and Tarjan (1986),
Juris Hartmanis and Richard E. Stearns(1993),
Mauel Blum(1995), Andrew Yao(2000), Rivest, Shamir,
Adleman(2002), Leslie Valiant(2010), Micali, Goldwasser(2012)

ネバンリンナ賞

Tarjan(82), Valiant(86), Razborov(90), Wigderson(94), Shor(98)
Sudan(02), Kleinberg(06), Spielman(10), Khot(14)

ユダヤ人が半数以上！

Non-US, Non-Israel: Mikkel Thorup, Ken-ichi Kawarabayashi
Under 40: Venkatesan Guruswami, Anupam Gupta
Ken-ichi Kawarabayashi

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[Zvi Galil](#) (46)
[Ran Raz](#) (45)

理論研究の世界： STOC, FOCS, SODA世界

ここで生き残るのは結構つらい！

→ 理論屋さんだけでやっていくのは結構つらいよ！

→ でも他の分野でも活躍できる！

でも... 数学なのに、評価される理由

1. 実用面での貢献をした研究者も多い
2. 理論が強い学生は、常に優良物件！
3. 他分野にも影響を与える研究者も多い！

Alon: アーベル賞を10年以内にとりそうな数学者

Leighton: Akamaiの創始者

Kleinberg...

Piotr Indyk: ICML'15 ベストペーパー

Ragahavan: Former Y! research director

S. Muthukrishnan: Databaseでもビッグネーム

Motowani: Brin, PageのPDメンター

C. Dvorak: Privacyでの超有名人

その他: 理論研究者が他分野で、論文をたくさん発表！

MY STOC PAPERS

1. Ken-ichi Kawarabayashi, Mikkel Thorup
Deterministic *Global Minimum Cut* of a Simple Graph in Near-linear Time
2. Ken-ichi Kawarabayashi and Stephan Kreutzer,
The *Directed Grid Theorem*
3. Ken-ichi Kawarabayashi and Anastasios(Tasos) Sidiropoulos
Beyond the Euler Characteristic: Approximating the Genus of General Graphs

90/323..

Deterministic Global Minimum Cut of a Simple Graph in Near-linear Time

By KK and Mikkel Thorup

Summary: not best paper, but

- ▶ Finding edge connectivity λ one of the most basic graph problems "Can cutting k edges disconnect my graph?". You can explain it to a child.
- ▶ Dates back at least 54 years within TCS, but concept probably pre-historic as splitting the enemy...
- ▶ Previous best deterministic $\tilde{O}(\lambda m)$ time by Gabow [1993].
- ▶ **This paper:** Near-optimal $\tilde{O}(m)$ time for classic graphs (simple graphs, not quasi/multi-graphs).
- ▶ Solution involves new structural understanding of simple graphs (not just "general" techniques).
- ▶ We find multi-graph \bar{G} with $\tilde{O}(n/\delta)$ vertices and $\bar{m} = \tilde{O}(n)$ edges preserving all non-trivial min-cuts of G .
- ▶ Apply Gabow to \bar{G} yields min-cut and cactus for G in $\tilde{O}(\delta \bar{m}) = \tilde{O}(m)$ time.
- ▶ Existence of \bar{G} was not known.

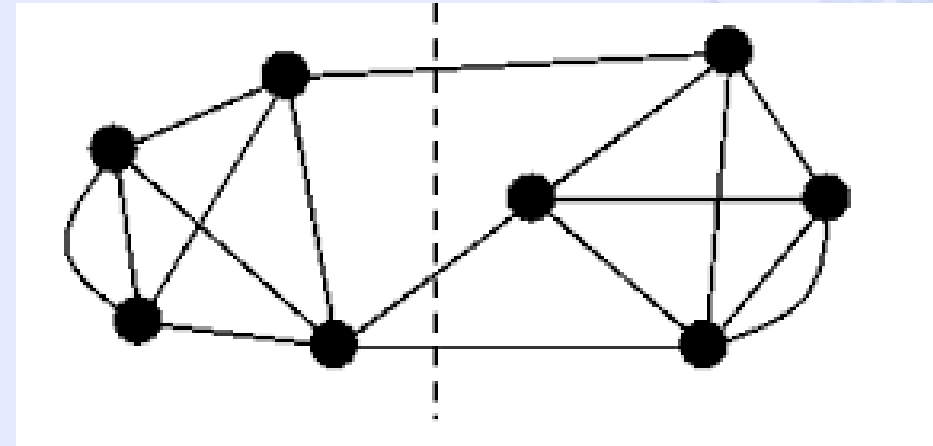
STOC'15 Result: Edge Connectivity

Graph Cut: One of the most famous problem

Partition G into (A,B) such that # of edges between A and B is as small as possible.

G : n vertices and m edges.

K : cut size

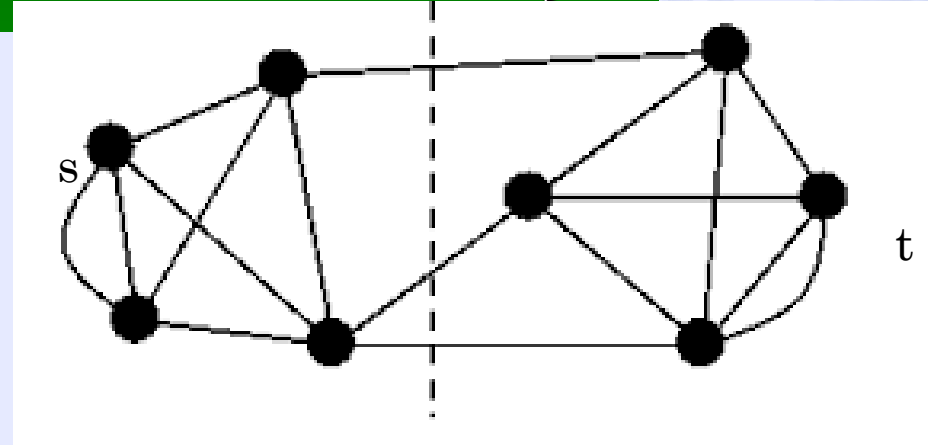


Race by many ``giant'' researchers in theory!

Ford and Fulkerson'56	$O(mn^2)$	←	Founder of Operation Research !
Even and Tarjan'75	$O(nm^{1.5})$	←	Turing prize winner
Karzanov and Timofeev'86	$O(n^3)$	←	Russian researchers
Hao and Orlin'92	$O(nm \log(n^2/m))$		Deterministic
Ibaraki and Nagamochi'92	$O(nm+n^2 \log n)$		Deterministic
Gabow'93	$O(km)$		Deterministic (for simple graphs)
Karger'94	$O(mn^{0.5})$		(Randomize, Monte Carlo, Random Contraction)
Karger and Stein'96	$O(n^2 \log^3 m)$		(Randomize, Monte Carlo)
Karger'99	$O(m \log^3 n)$		(Randomize, Monte Carlo, Tree Packing)
KK and Thorup(STOC'15)	$O(m \log^{10} n)$		Deterministic! (for simple graphs)

Min-Max Theorem(Ford and Fulkerson)

Given s and t ,
Max # of edge-disjoint paths (flows)
= Min # of edge cut s and t



Race by many ``giant'' researchers in theory!

Ford and Fulkerson'56	$O(mn^2)$	←
Even and Tarjan'75	$O(nm^{1.5})$	←
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KK and Thorup(STOC'15)	$O(m \log^{10} n)$	

Founder of Operation Research !

Turing prize winner

Russian researchers

Deterministic

Deterministic

Deterministic (for simple graphs)

(Randomize, Monte Carlo, Random Contraction)

(Randomize, Monte Carlo)

(Randomize, Monte Carlo, Tree Packing)

Deterministic! (for simple graphs)

Random Algorithms: Two kinds

- ◆ **Monte Carlo Algorithm:**

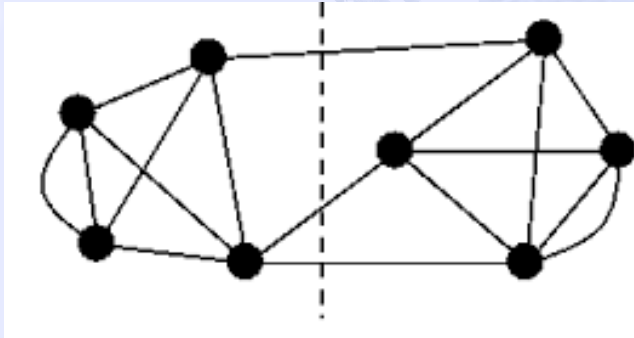
Guarantee the time complexity, and with high probability, the answer is correct.

Karger's mincut: NO WAY to check that the cut is min...

- ◆ **Las Vegas Algorithm:**

Guarantee the correctness, but with small probability, time complexity takes forever..

Why “simple” graphs?



Key Observation 1:

Let C be a “minimum” cut (of order at most k). Either

1. One side is trivial (note that min degree is at least k), or
2. Both sides contain at least $k-1$ vertices..

Our luck:

Can assume that $k > \log^3 m$ (otherwise, we can get a min cut algorithm with running time $O(km)$ by Gabow, and done.)

In 2, we get a “sparse” cut (i.e., low conductance cut)!

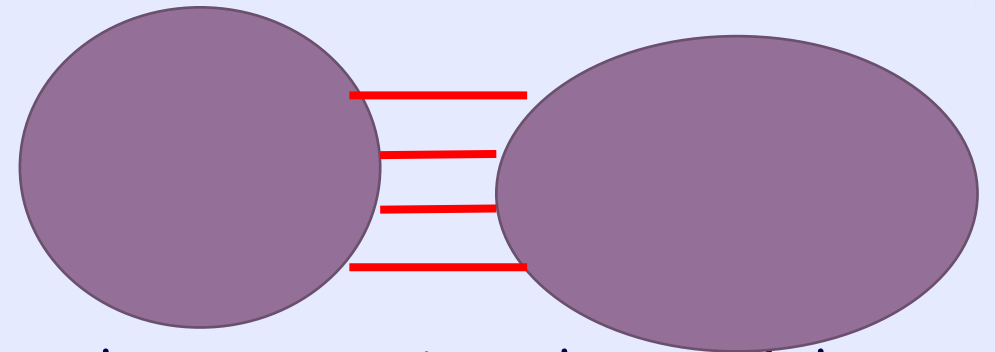
Log n is a “magic” number... → **KEY IDEA! Keep applying sparsest cut!**

Contractions!

- ◆ **Observation 2:**

Q: a subgraph s.t. each vertex in Q has degree at least $k/2$ in Q. **If no sparsest cut of conductance $k/4$ in Q \rightarrow Q is contractible**
(preserving all min nontrivial cuts)!

Note: $k > \log^3 n$. So conductance is small..



The idea:

Find contractible subgraphs Q' such that after the contraction, the graph has at most m/k edges (but all nontrivial cuts are there).

\rightarrow By Gabow's, we get an $O(m)$ algorithm.

How to find contractible subgraphs? Repeatedly cutting sparsest cuts!

\rightarrow When we stuck, we get "contractible" subgraphs!

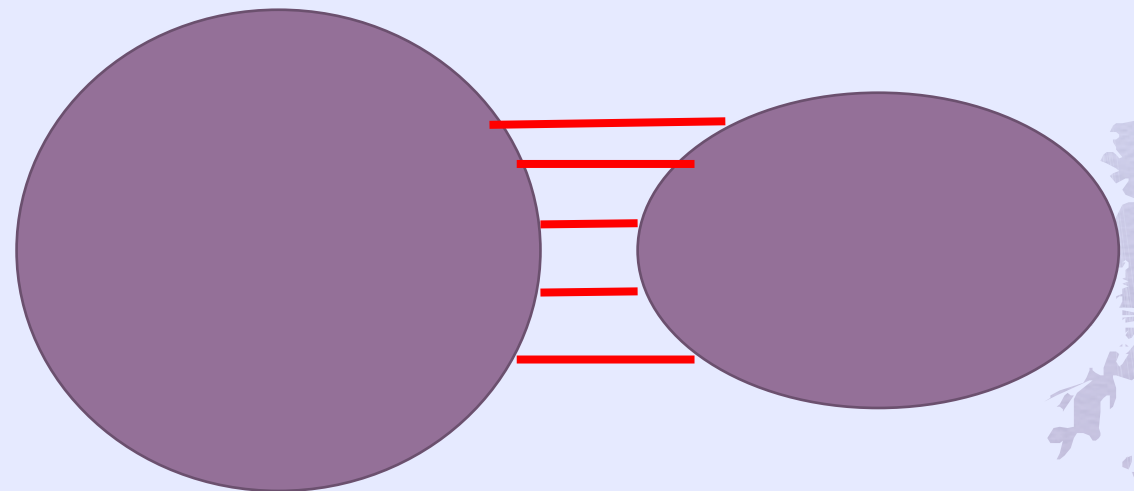
Some technical components for Sparse cut:

- ◆ PageRank! (starting from one vertex) & Sparsest cut!
(Andersson, Chung, Lang)

Key Observation:

Let G be a simple graph with min degree k . If min-cut W is not trivial, then # of cut edges of W is at most factor $2/k$ of # of edges in components.

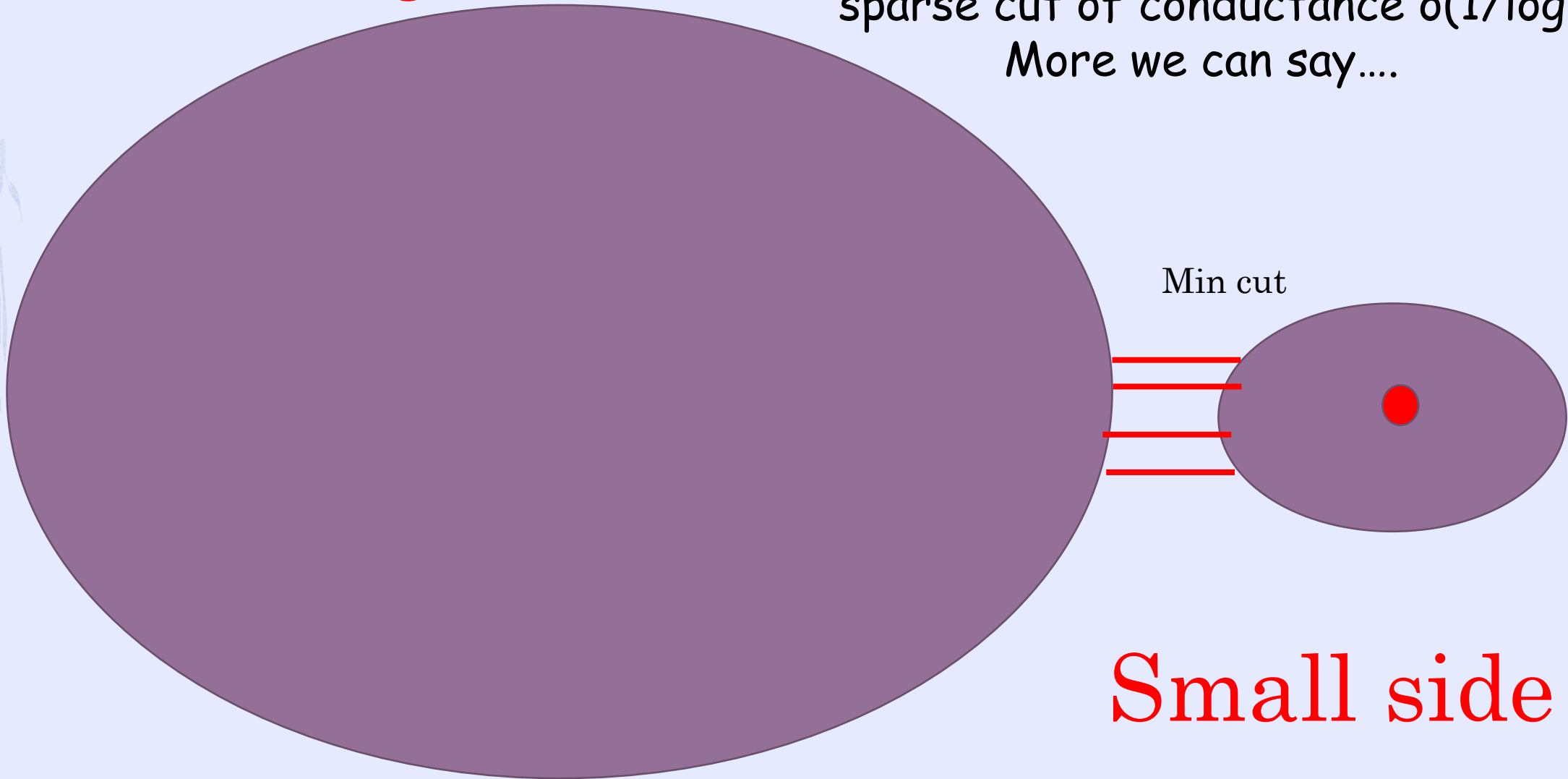
- we can use PageRank to find a sparsest cut W' (with error factor $\log n \ll k$)!
- # of cut edges for W' is at most factor $(2 \log n/k)$ of # of edges in W' in $O(m)$!
 $k > \log^5 m$



Big Side G-W

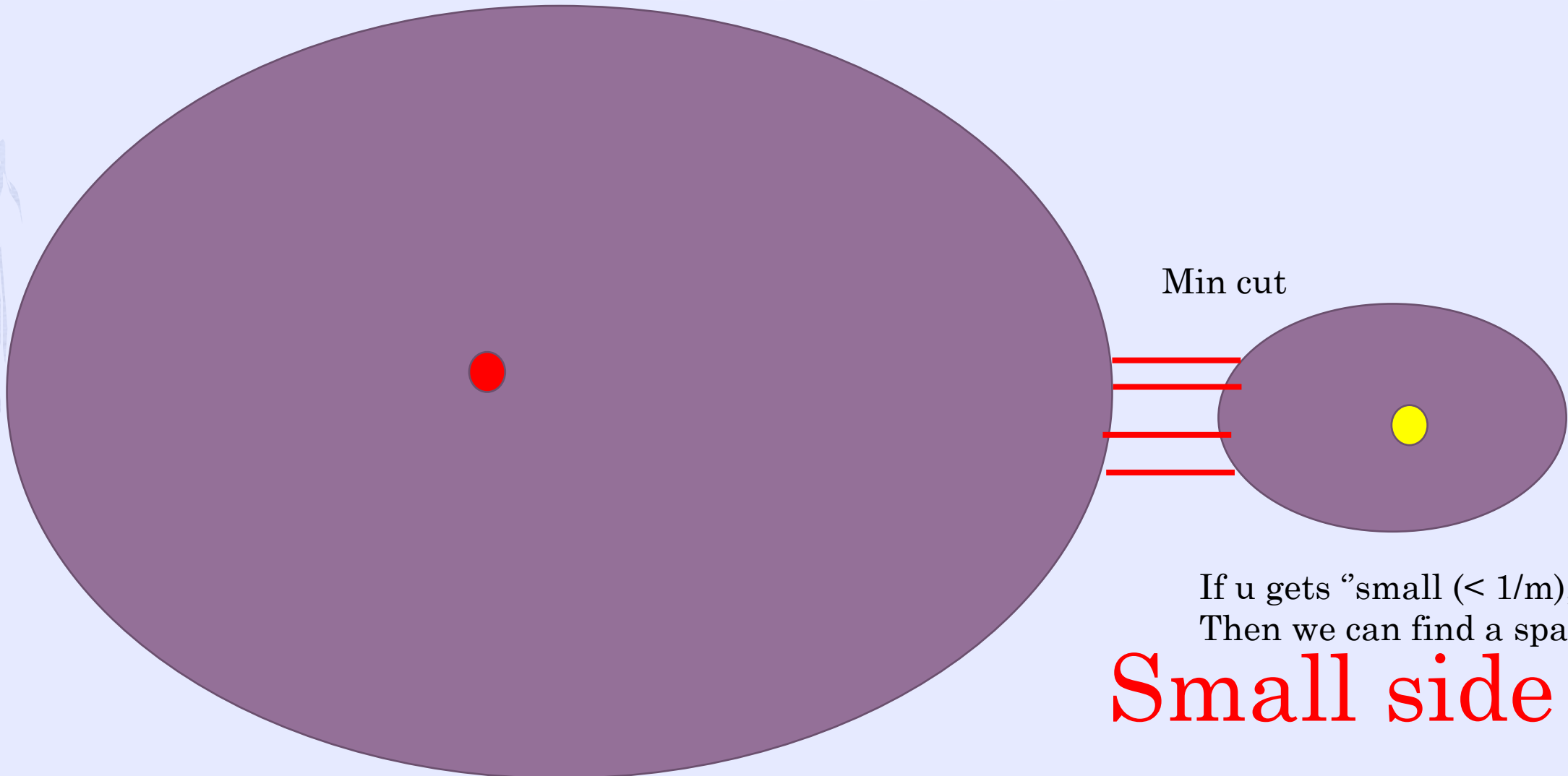
If we can ``correctly'' guess a vertex v in W and start PageRank from v , we can get a sparse cut of conductance $o(1/\log n)$.

More we can say....



Small side W

End Game!

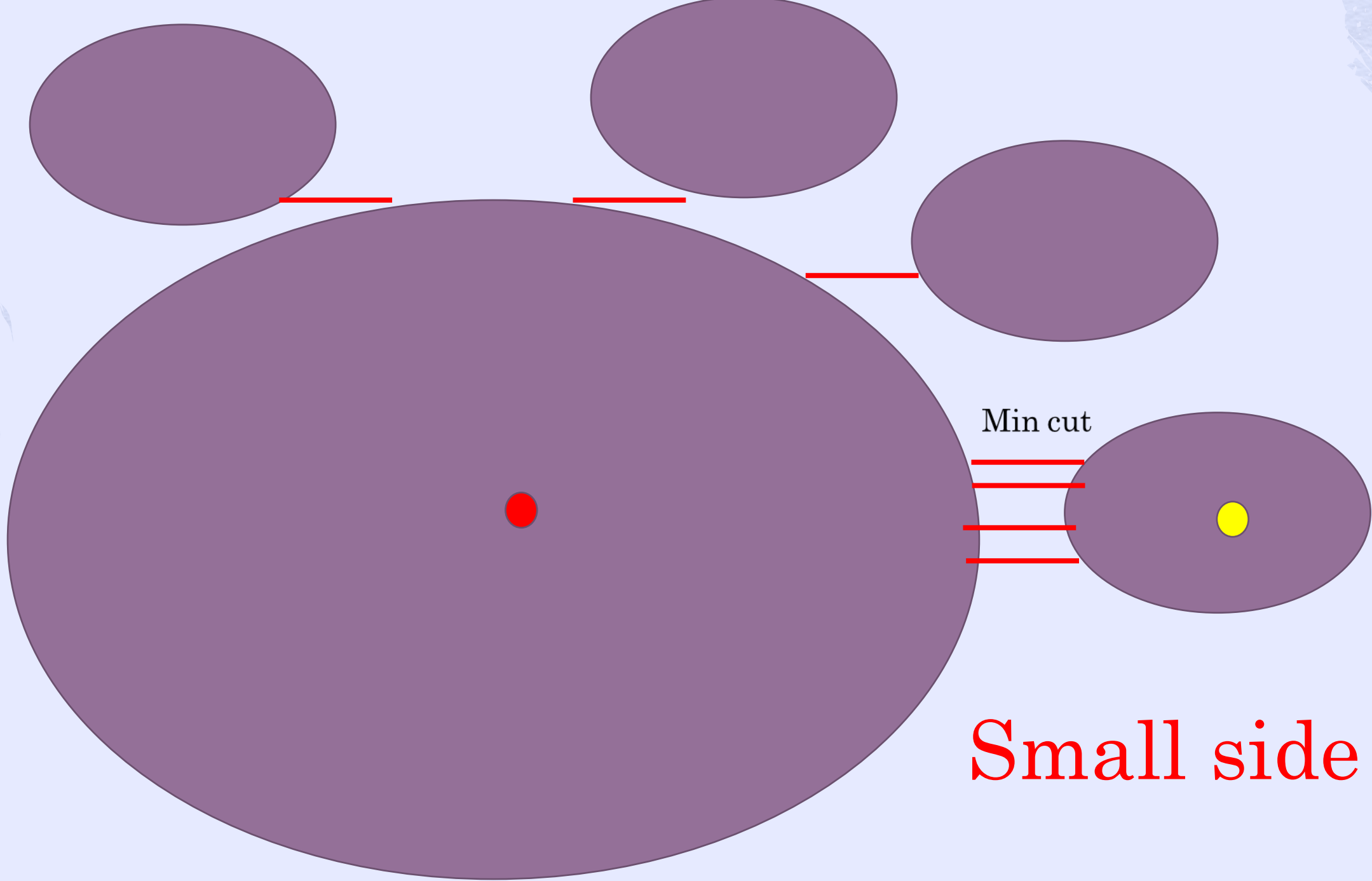


Min cut

If u gets "small" ($< 1/m$),
Then we can find a sparse cut!

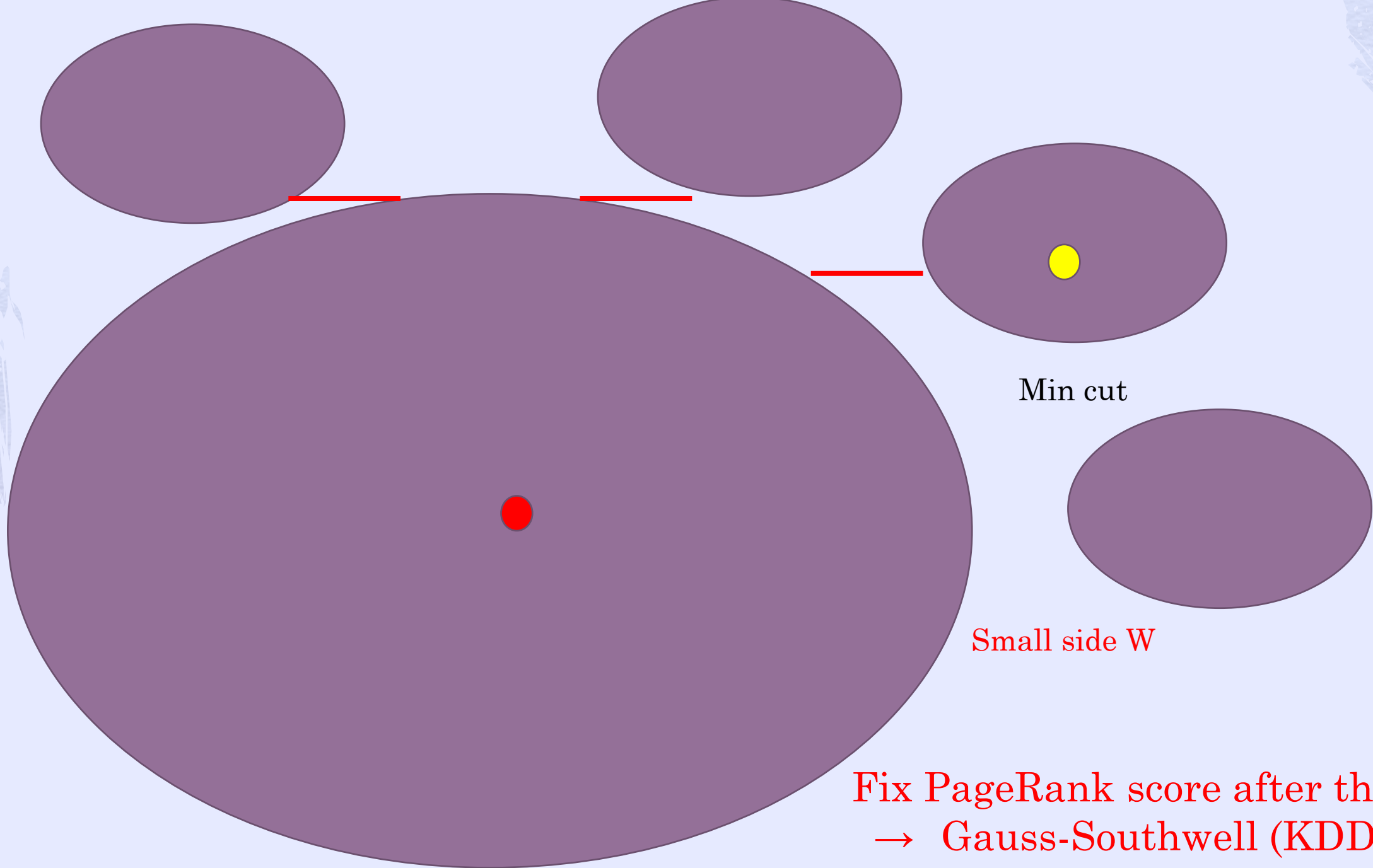
Small side W

Big Side G-W



Min cut

Small side W



Fix PageRank score after the cut of W

→ Gauss-Southwell (KDD'15)!

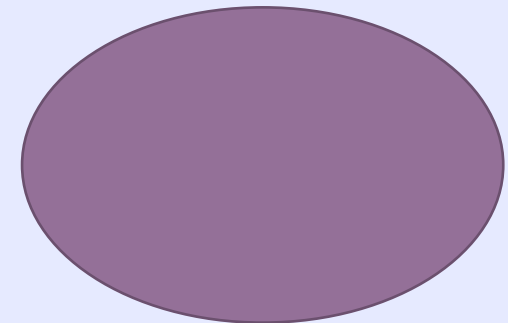
→ Time complexity: $O(|W|)$

Finally

- ◆ If we cannot find low-conductance (sparse) cut in W , then W is contractible.
- ◆ **The point:**
Only apply $O(\log n)$ iterations to the small side $W \rightarrow$ average degree is $k - O(\log^2 n) \gg k/2 \dots$
- ◆ Can do each “small” component separately..
 \rightarrow spend only $O(m)$ in total. Then we can find contractible subgraphs!

In the end, we have a graph with $O(m/k)$ edges.

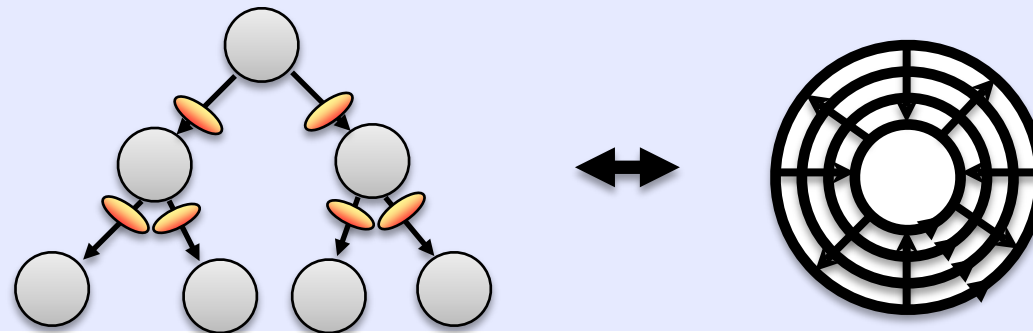
Well, cheating.....



The Directed Grid Theorem

Ken-ichi Kawarabayashi, NII, Tokyo
Stephan Kreutzer, TU Berlin

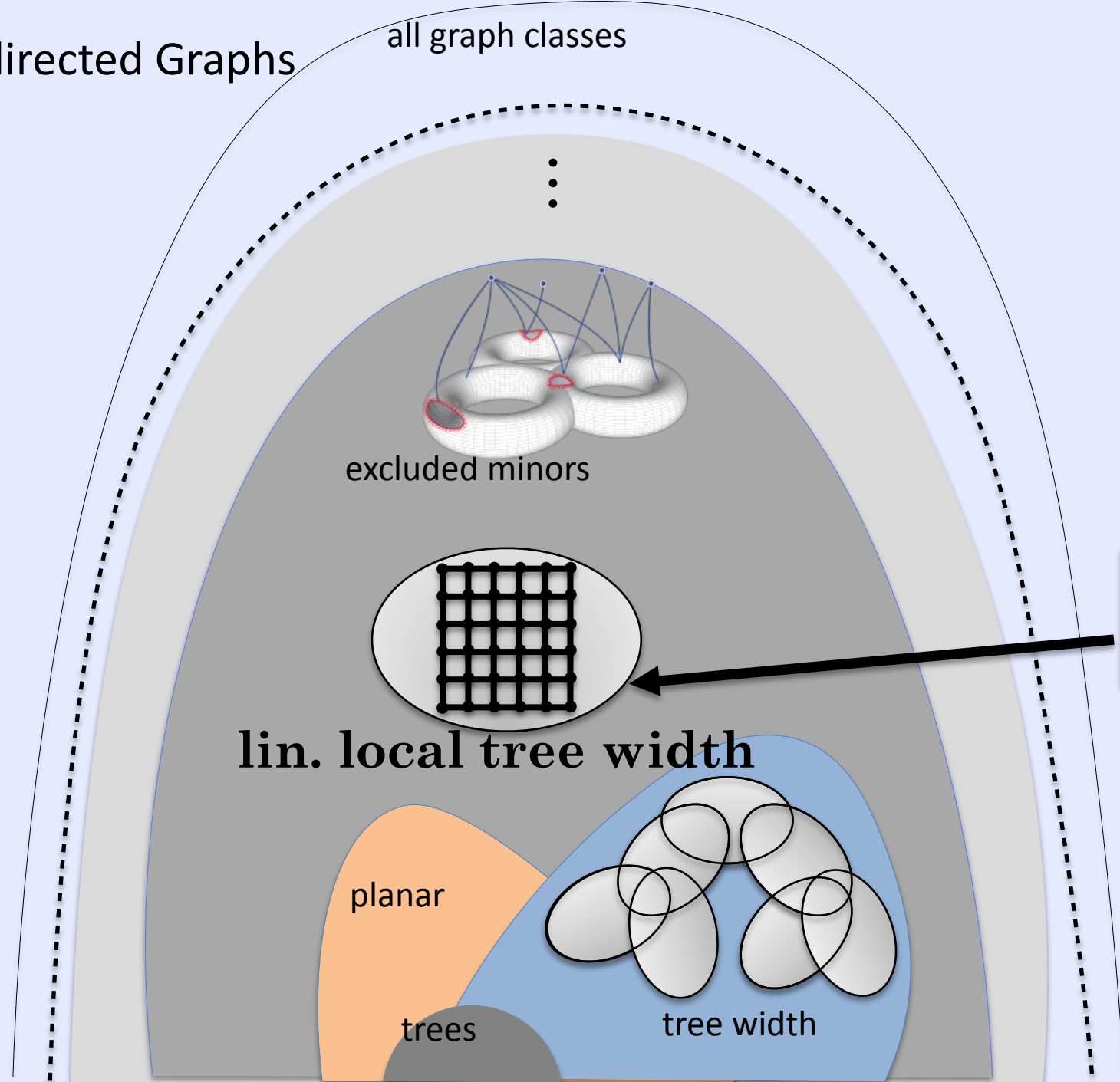
(ICALPのほうが詳しい)



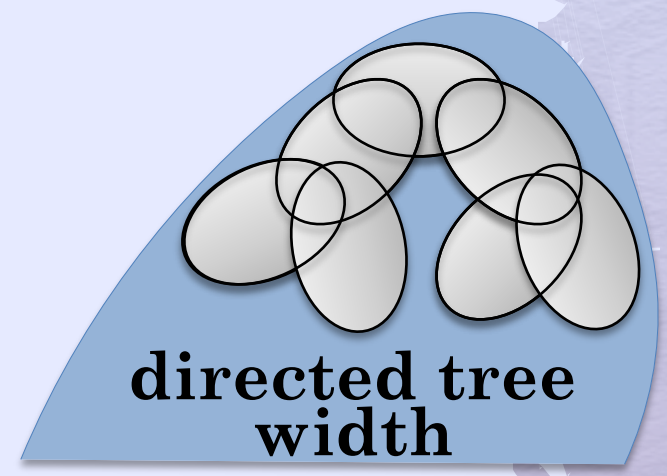
Undirected Graphs

all graph classes

Directed Graphs



The Grid Theorem



The main motivation of this talk

WHAT ABOUT DIGRAPHS?

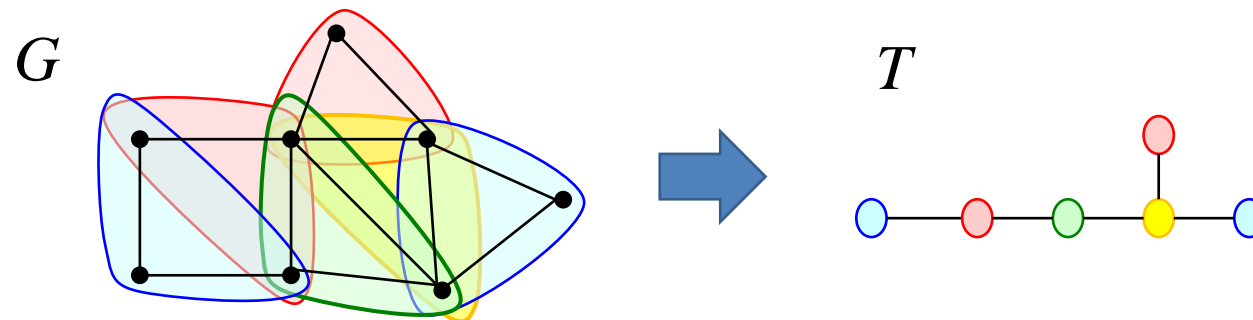
Can we do the same thing?

Warning: Not really known much.

Purposes of this talk: report some progress
future direction
and many open questions

Treewidth

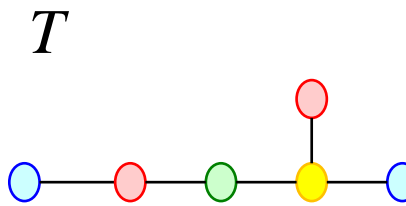
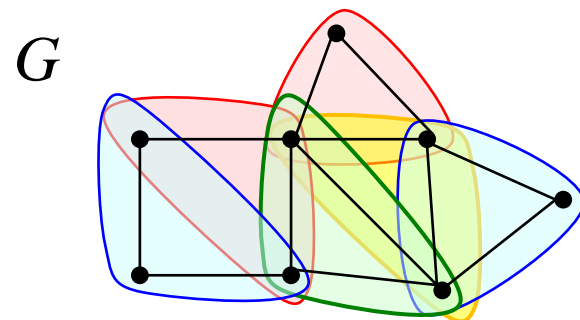
- (T, \mathbf{W}) is a **tree decomposition** of $G = (V, E)$
 - T is a tree, $\mathbf{W} = \{W_t \subseteq V \mid t \in V(T)\}$
- ↔
- $\bigcup_{t \in V(T)} W_t = V$, and every edge has both ends in some W_t
 - If t lies on the path in T between t' and t'' , then $W_t \subseteq W_{t'} \cap W_{t''}$



Tree-width:
How close to a Tree
Width=1 \Leftrightarrow Tree

Treewidth

- (T, \mathbf{W}) is a **tree decomposition** of $G = (V, E)$
 - T is a tree, $\mathbf{W} = \{W_t \subseteq V \mid t \in V(T)\}$
- ↔
 - $\bigcup_{t \in V(T)} W_t = V$, and every edge has both ends in some W_t
 - If t lies on the path in T between t' and t'' , then $W_t \subseteq W_{t'} \cap W_{t''}$
- **width** of $(T, \mathbf{W}) := \max_t |W_t| - 1$
- **treewidth** of $G := \min.$ **width** over all tree decompositions



treewidth = 2

Tree-width:

How close to a Tree
Width=1 \Leftrightarrow Tree

Importance of Tree-width

- (**Algorithm**) Many hard NP-hard problems can be solved in linear time.
- (**Algorithm**) Fixed parameter-tractability
- (**Graph Theory**) Great tools in Structure Graph Theory
- (**Logic**) Courcelle's theorem: every graph property that is definable in MSO_2 can be decided in linear time for graphs of bounded tree-width!
- (**General**) AI, Database, Data mining, Social Network, Programming Language etc.

Treewidth and Grid minor (most important result in treewidth..)

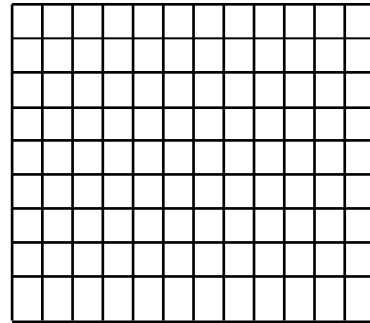
Thm (Robertson-Seymour, 1986) The grid minor theorem

For any r , there exists an integer $f(r)$ s.t. for any graph G , either

- treewidth of G is $\leq f(r)$
- G has an $r \times r$ -grid-minor

Known results

- $f(r) = 2^{O(r^5)}$, $f(r) = \Omega(r^2 \log r)$ (Robertson-Seymour-Thomas [1994], Diestel et al. [1999])



Tree-width r

G contains an H -minor $\rightarrow H$ can be obtained from G by deleting edges and contracting edges

Treewidth and Grid minor (most important result in treewidth..)

Thm (Robertson-Seymour, 1986) The grid minor theorem

For any r , there exists an integer $f(r)$ s.t. for any graph G , either

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Known results

- $f(r) = 2^{O(r^5)}$, $f(r) = \Omega(r^2 \log r)$ (Robertson-Seymour-Thomas [1994], Diestel et al. [1999])
- G : planar $f(r) = O(r)$ (Robertson-Seymour-Thomas [1994])
- G : bounded genus $f(r) = O(r)$ (Demaine, Fomin, Hajiaghayi, Thilikos, [2005])
- G : H -minor-free $f(r) = c_H \cdot r$ (Demaine-Hajiaghayi [2008])
- G : $K_{3,k}$ -minor-free $f(r) = 20^{4k} \cdot r$ (Demaine-Hajiaghayi –KK [2009])

Treewidth and Grid minor (most important result in treewidth..)

Thm (Robertson-Seymour, 1986) The grid minor theorem

For any r , there exists an integer $f(r)$ s.t. for any graph G , either

- treewidth of G is $\leq f(r)$
- G has an $r \times r$ -grid-minor

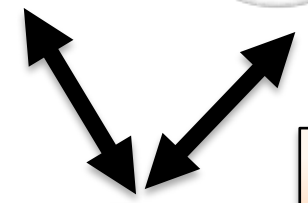
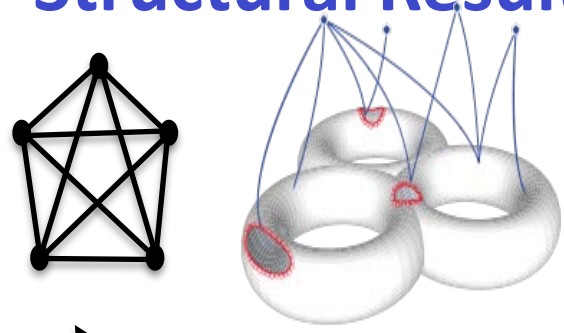
Known results

- $f(r) = 2O(r^5)$, $f(r) = \Omega(r^2 \log r)$ (Robertson-Seymour-Thomas [1994], Diestel et al. [1999])
- $G : H$ -minor-free $f(r) = |V(H)|^{O(E(H))} \cdot r$ (Kawarabayashi-Kobayashi [2012])
- $G : \text{general}$ $f(r) = 2^{O(r^2 \log r)}$ (Kawarabayashi-Kobayashi, Leaf-Seymour [2012])
- $G : \text{general}$ $f(r) = r^{O(1)}$ (Chekuri-Chuzoy [2013])

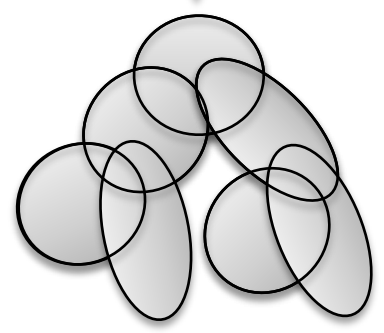
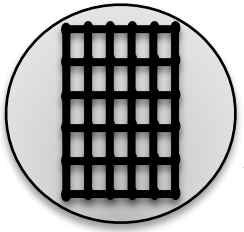
Importance of the grid minor theorem:

1. Fixed Parameter Tractability (FPT),
bidimensionality (Large vertex cover..)
2. Logic (FOL etc..)
3. Combinatorial Optimization (Routing..)

Structural Results



The Grid Theorem



all graph classes

⋮

excluded minors

planar

trees

tree width

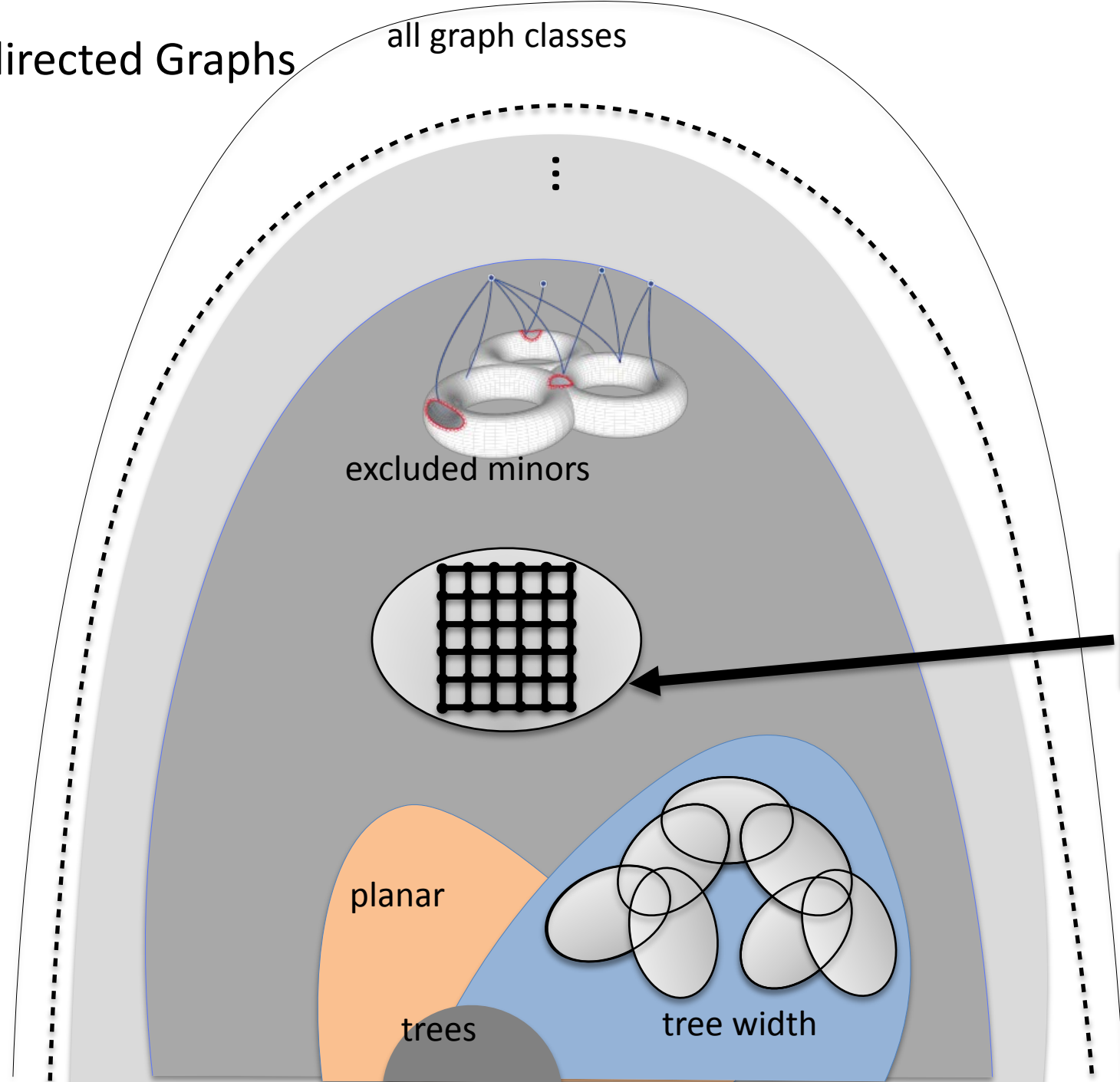
structural decomposition (Graph Minor)

dynamic programming

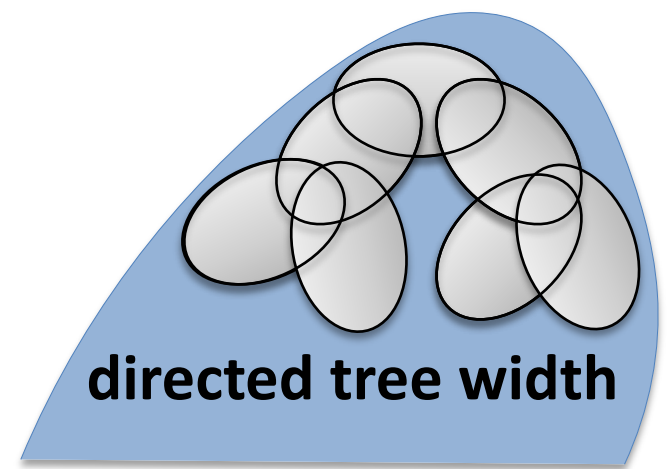
Undirected Graphs

all graph classes

Directed Graphs



The Grid Theorem



What about digraphs?

Seems much harder!

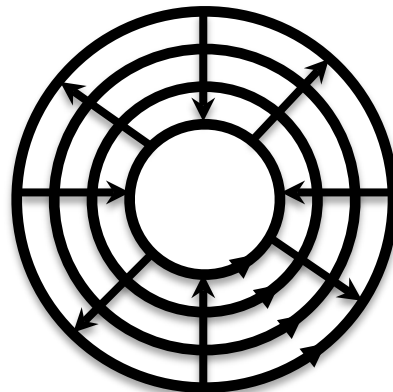
Q.1 What is "tree-width" for digraphs?

Q.2 What is "minor" for digraphs?

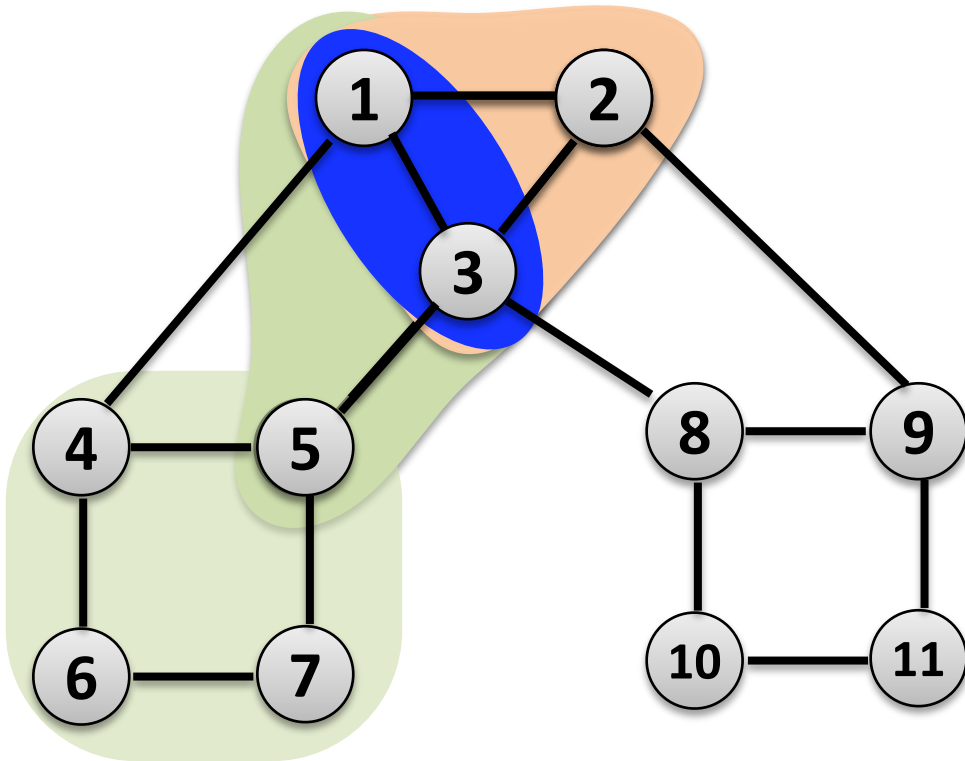
What is grid minor for digraphs?

What should be "grid"?

1. Many cycles!
2. Globally connected

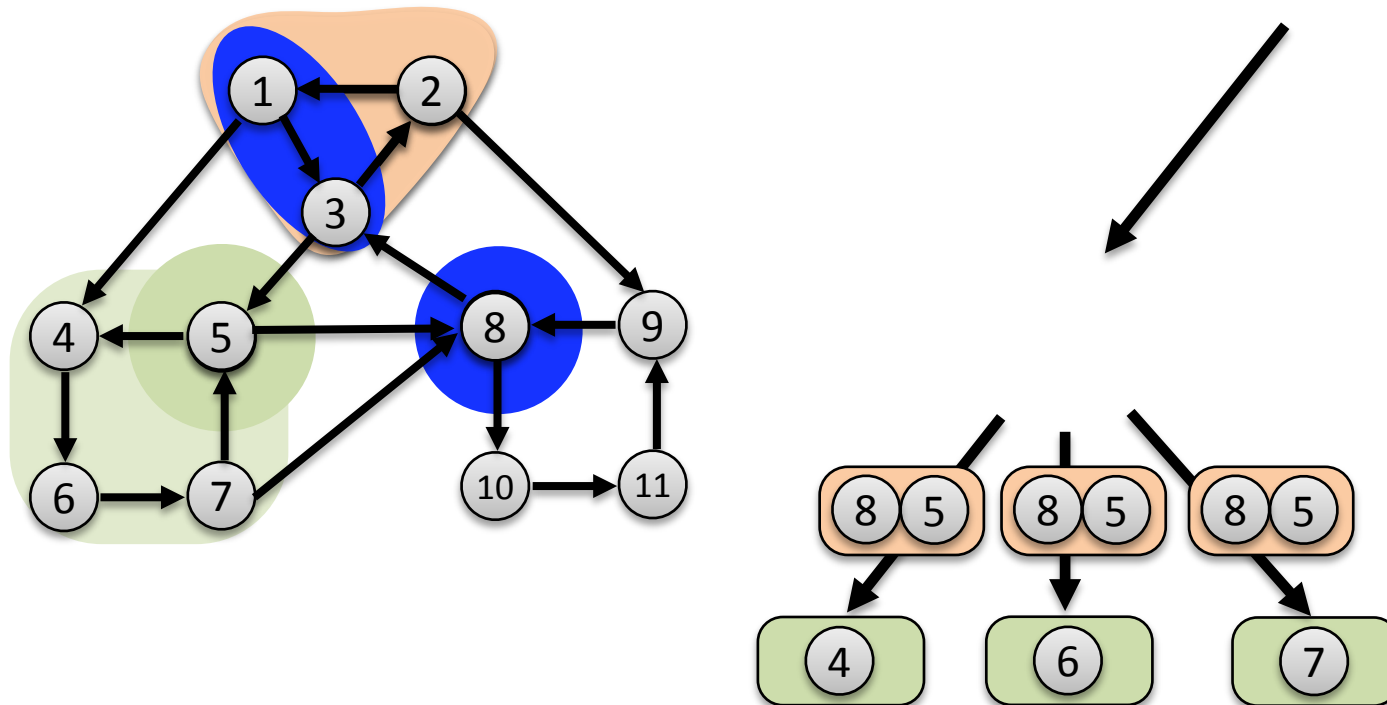


Tree-width for undirected graphs

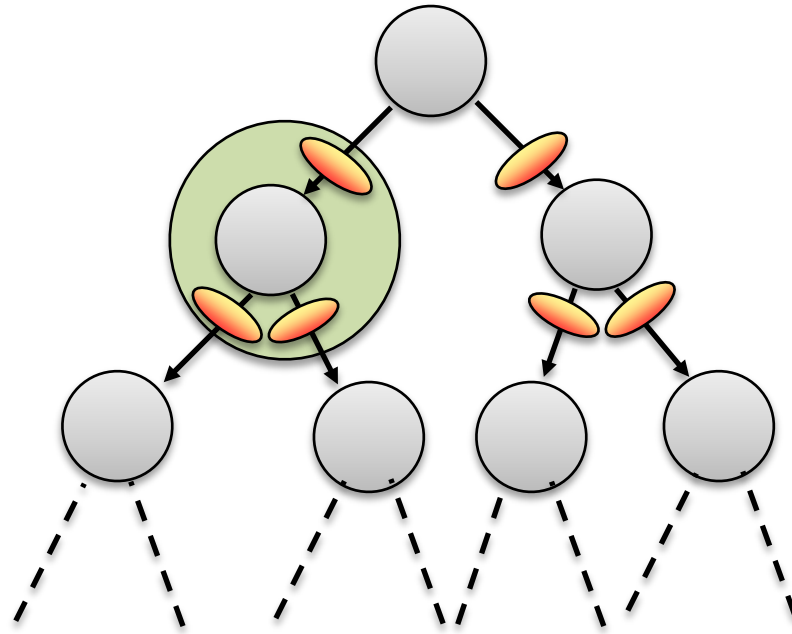


Q1. What is Directed Tree-Width?

- Defined analogous to undirected tree width but based on strong separators



Directed Tree Width



Directed tree-width:
How close to a DAG
(directed acyclic graph)
Width=1 \Leftrightarrow DAG

Definition.

(Reed'97) (Johnson, Robertson, Seymour, Thomas 01)

Directed tree decomposition of G : triple (T, β, γ)

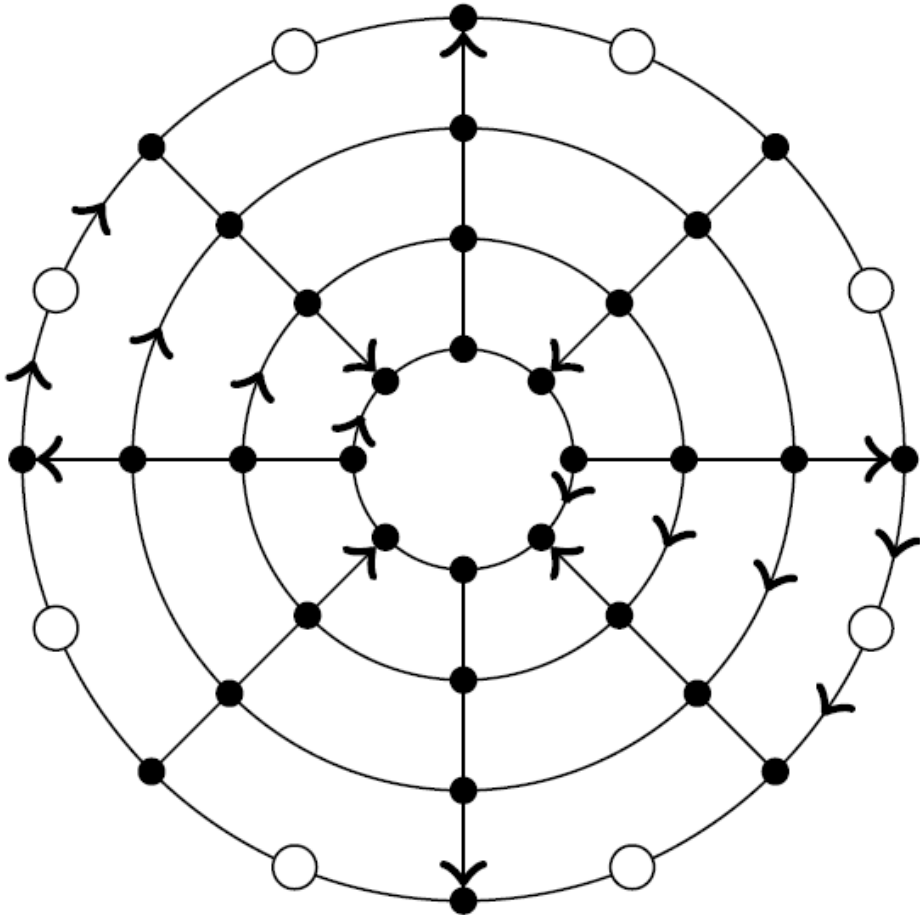
- bags $\beta(t) \subseteq V(G)$ form a partition of the vertex set of G
- for all $e=(s,t) \in E(T)$: $\gamma(e) \subseteq V(G)$

$$\beta(T_t) := \bigcup \{ \beta(t') : t \leq_T t' \} \text{ strong component of } G - \gamma(e)$$

The **width** is the minimum of $|\beta(t) \cup \bigcup_{e \sim t} \gamma(e)|$ over all $t \in V(T)$.

CONJECTURE (Alon, Reed, Robertson, Seymour, Thomas, 1995)

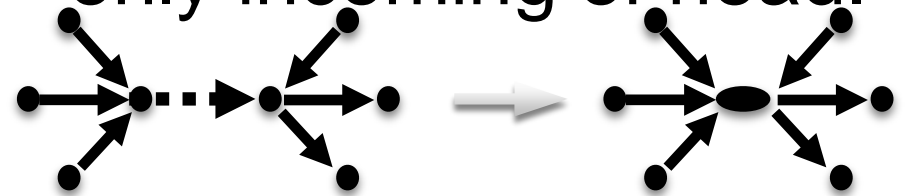
There is a function f such that every digraph of tree-width at least $f(k)$ has a cylindrical $k \times k$ grid minor.



Directed Grid should be

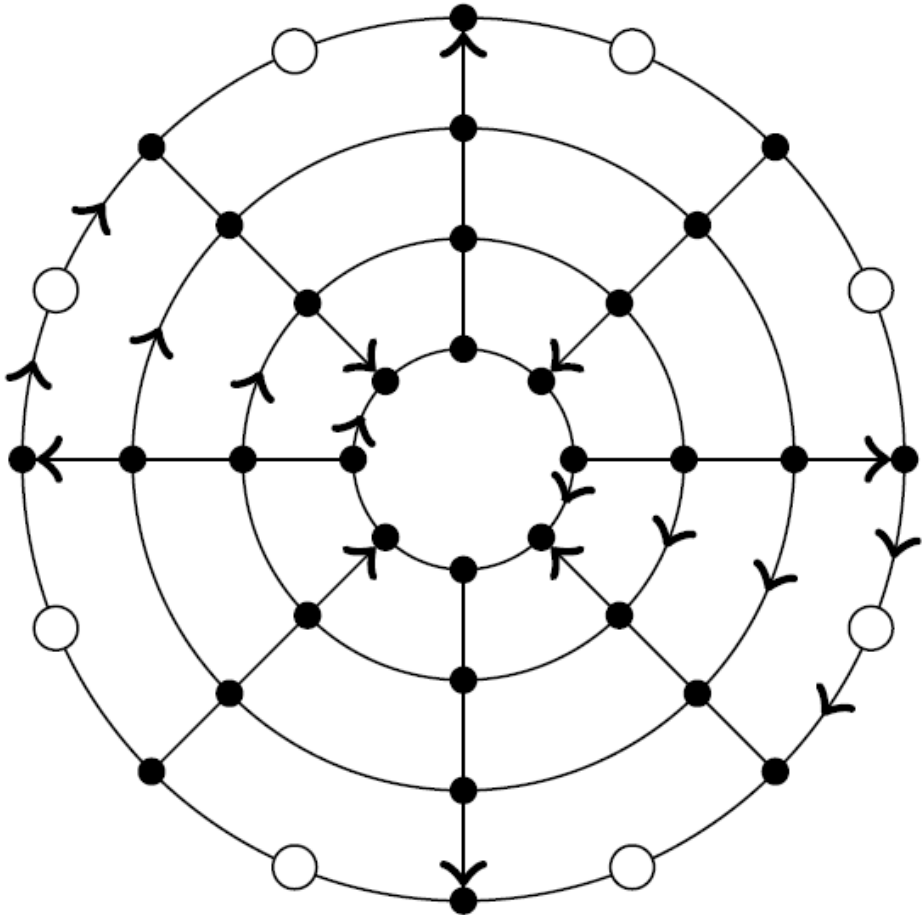
1. Many cycles
2. Globally connected
3. Directed tree-width at least k

Minor: contract an edge if only outgoing of tail or only incoming of head.



CONJECTURE (Alon, Reed, Robertson, Seymour, Thomas, 1995)

There is a function f such that every digraph of tree-width at least $f(k)$ has a cylindrical $k \times k$ grid minor.



History:

True for **planar graphs**

(Johnson, Robertson, Seymour, Thomas, 99)

True for **minor-closed**

(KK& Kreutzer, SODA'14)

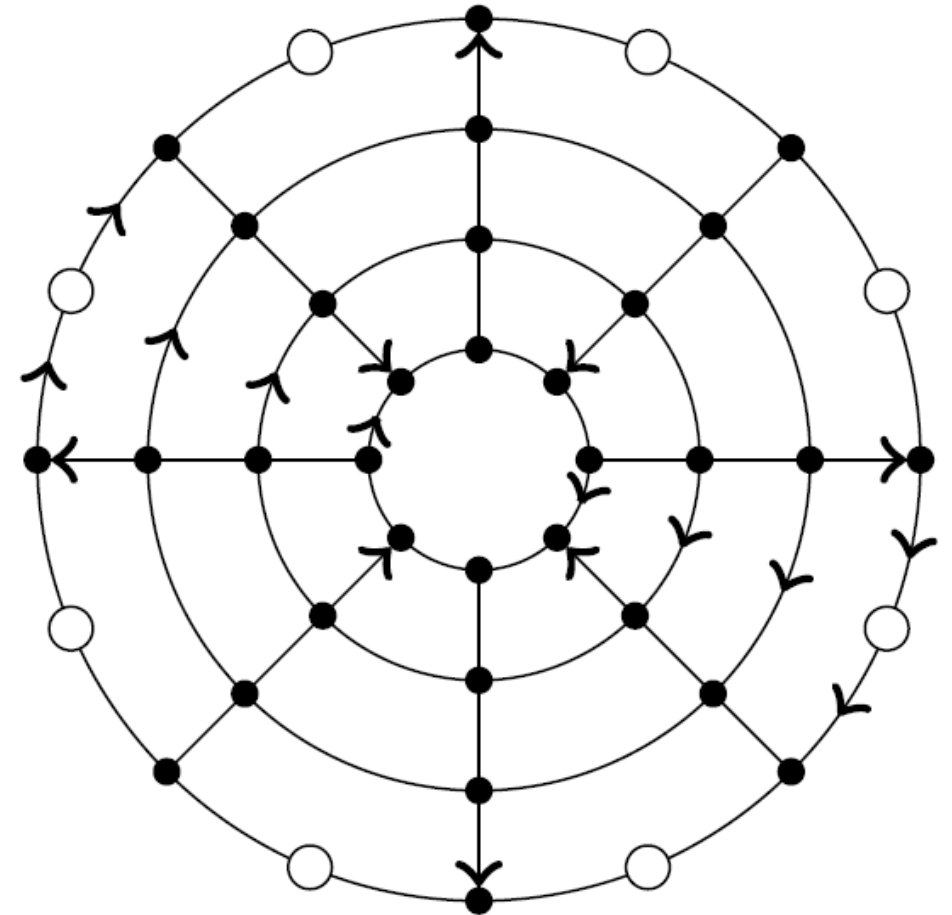
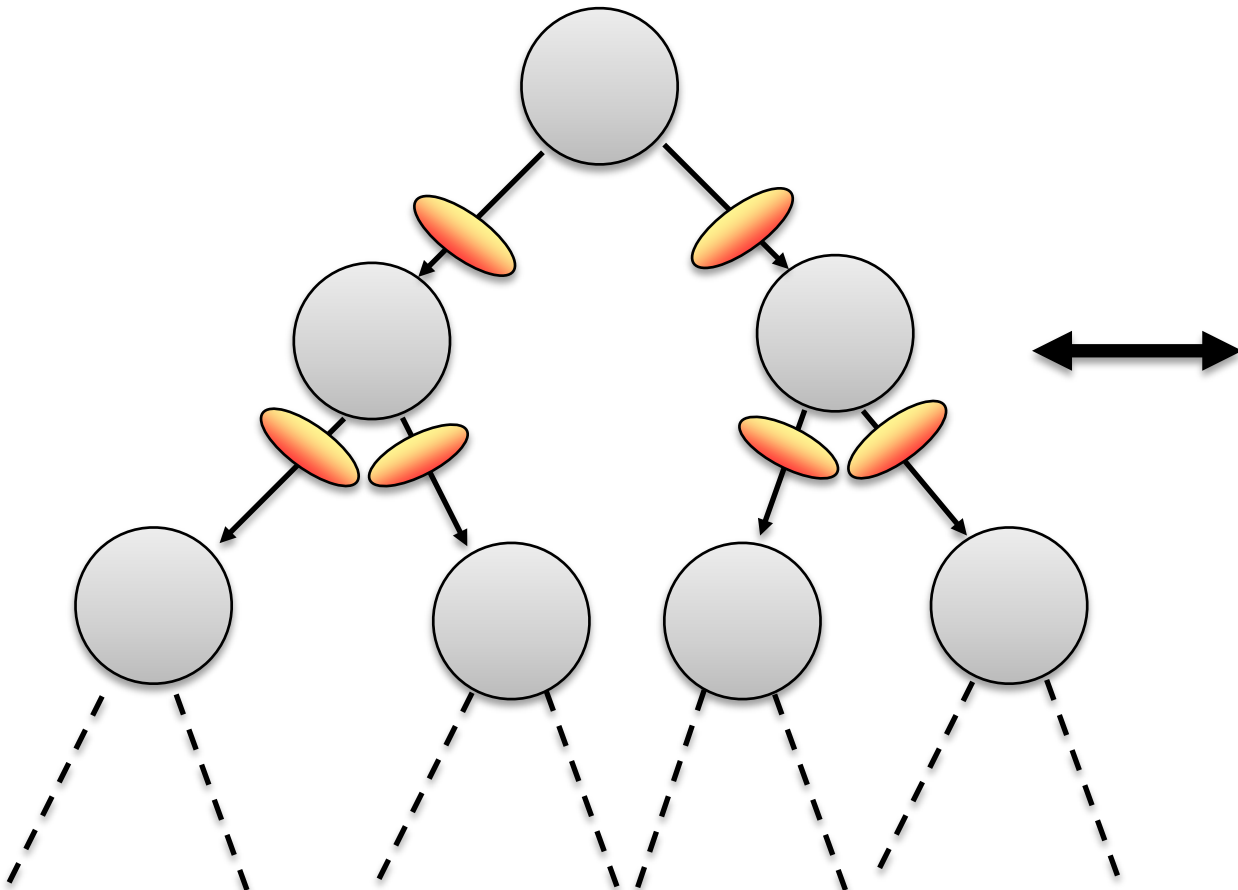
True if "half-integral"

(KK&Kreutzer'STOC'14)

General case?? -> **This is our main result!**

Main Theorem (KK and Kreutzer, STOC 2015)

There is a function f such that every digraph of tree-width at least $f(k)$ has a cylindrical $k \times k$ grid minor. Proof > 50 pages....



Undirected Graphs

all graph classes

excluded minors

planar

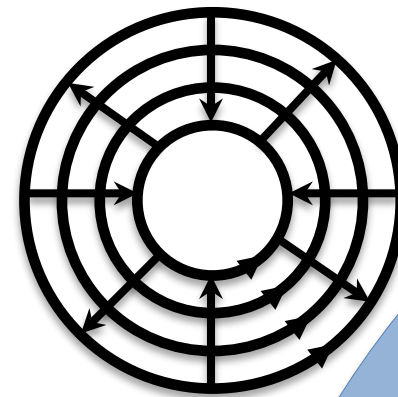
trees

tree width

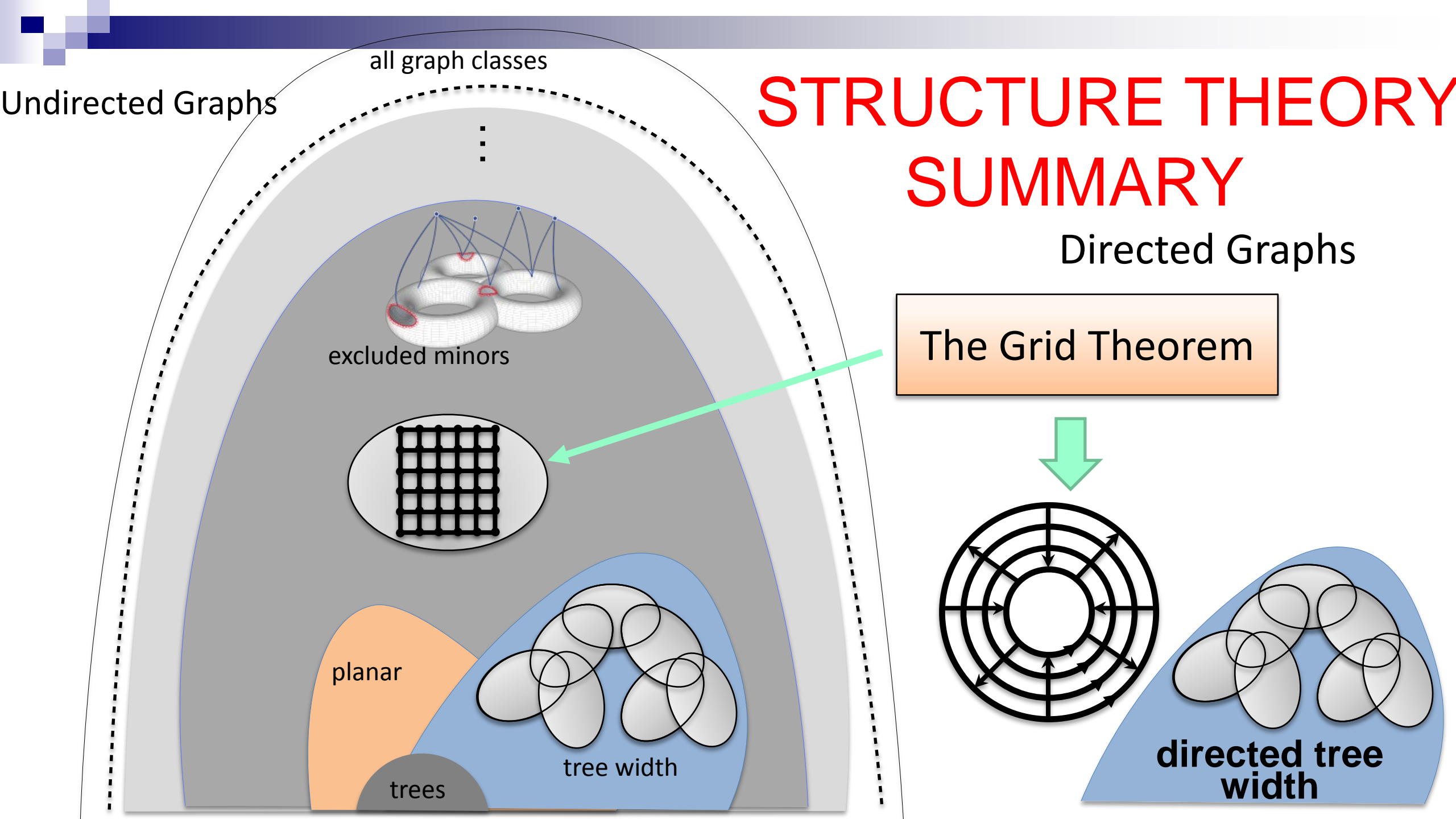
STRUCTURE THEORY SUMMARY

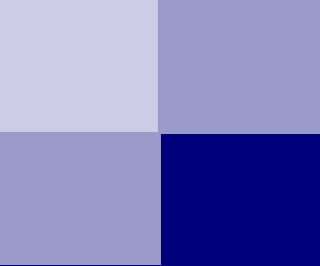
Directed Graphs

The Grid Theorem



directed tree width





Beyond the Euler genus: Approximating the genus of general graphs

Ken-ichi Kawarabayashi

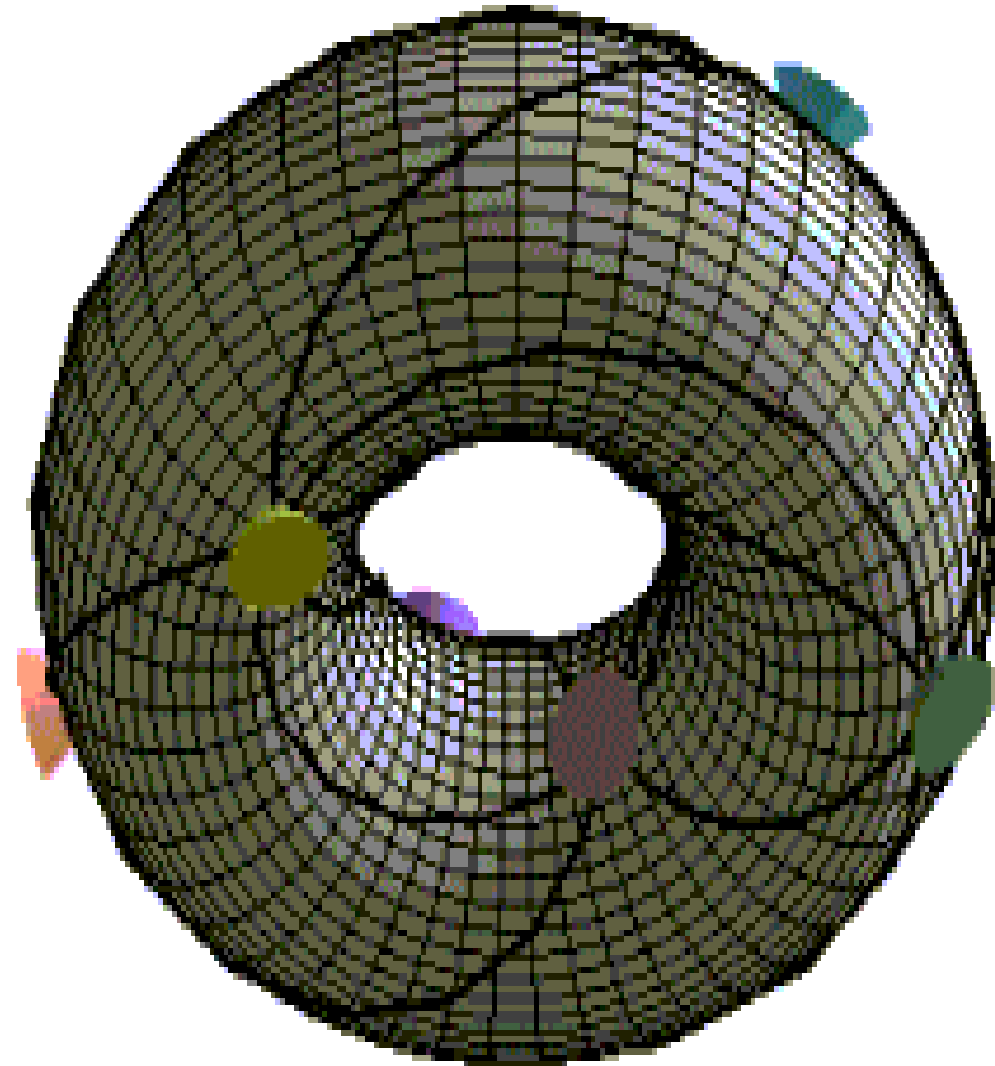
NII, Tokyo

Joint work with Anastasios(Tasos) Sidiropoulos

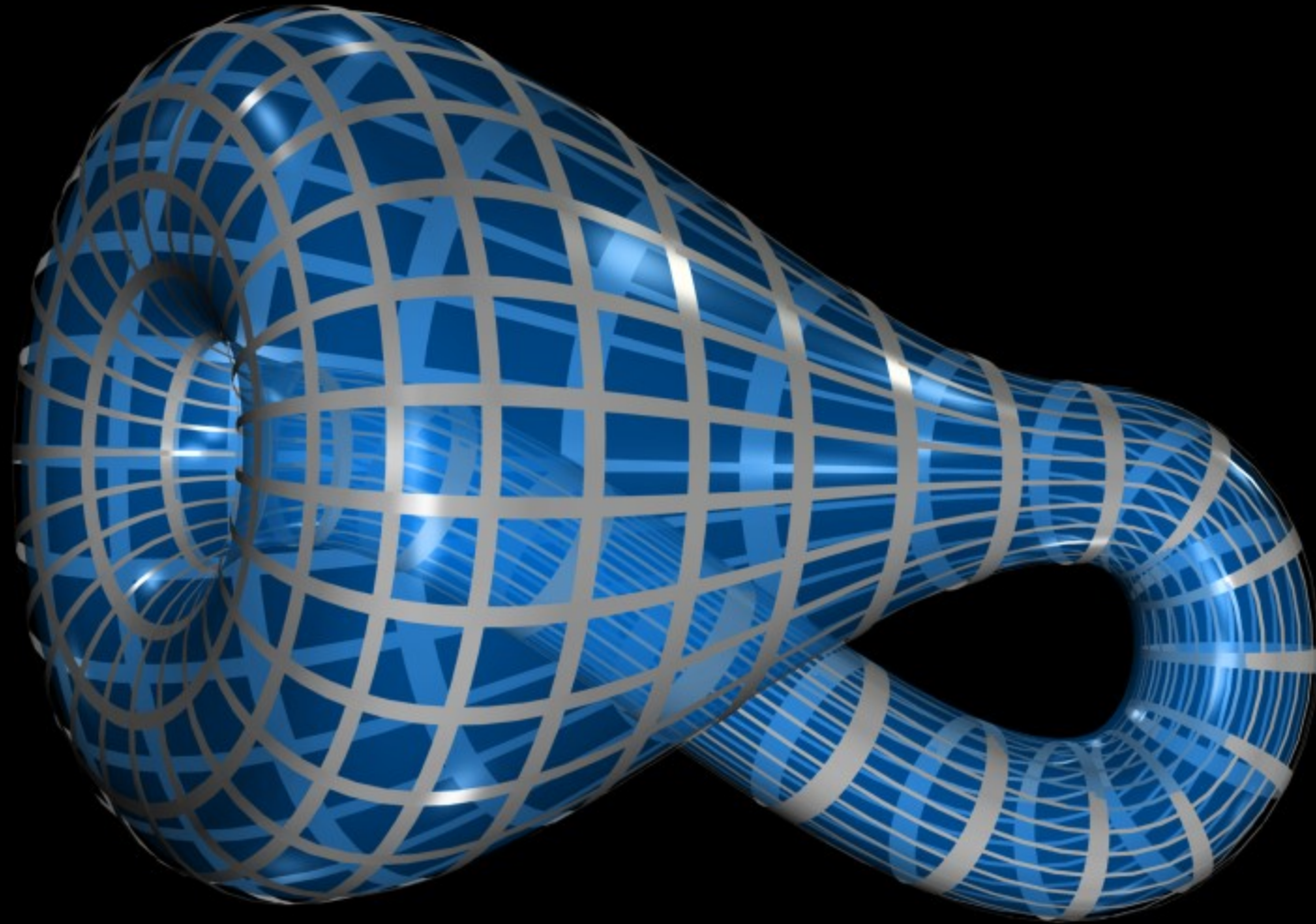
To appear in STOC'15

Surface

- By a surface, we mean a compact 2-dimensional manifold.
- We consider plane with crosscaps or handles.
- Plane + crosscap: Projective planar.
- Plane + handle : Torus
- Plane + two crosscaps : Klein bottle



Klein Bottle



Embedding into a surface

- Embed graph in a surface with no edge crossings.
- Minor-closed.
- Euler genus of complete graphs and complete bipartite graphs are known (Heawood 1890!)
- But very little about other graphs....

- **Problem(One of original Garey-Johnson)**

What is the genus of a given graph? Can you compute?

- **NP-complete** to determine Euler genus of a given graph. (**Thomassen, 1988**)

Embedding into a surface

- For fixed Euler genus g , $O(n^g)$ algorithm. (Filotti, Miller and Reif, 1990)
- $O(n^3)$ algorithm. (Robertson and Seymour, 1995)
- Linear time algorithm. (Mohar, 1997)
- Mohar's algorithm gives either an **embedding** in the surface of Euler genus k or an **obstruction** (One of minimal forbidden minors.)

New Algorithm(FOCS'08)

- With B. Reed and B. Mohar
- We get another linear time algorithm.
- Better in the following sense:
 1. Proof < 25 pages (Mohar's proof > 100 pages)
 2. Hidden constant is better. But 2^g

Approximation Algorithm

- No way to compute if g is bigger than $\log n$.
- NP-hard to decide g (**Thomassen**).
- Trivial approximation: $O(n/g)$.
Euler Formula: $|V| - |E| + |F| = 2 - g \rightarrow O(n/g)$ -approximation.
- Essentially **nothing** is known between NP-hardness and Trivial bound!
- No hardness result is known! (**$O(1)$ is NOT ruled out**)

Our main result: First “non-trivial” approximation

Theorem: Given a graph G and an integer g , in P , we can either

1. Correctly determine that G cannot be embedded in a surface of Euler genus g , or
2. Output an embedding of G in a surface of Euler genus $g^{200} \log^{150} n$.

Corollary. $O(n^{1-c})$ -approximation algorithm to compute Euler genus

For some small (but absolute constant) c .