Extension to 3-colorable triangulation

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Joint work with

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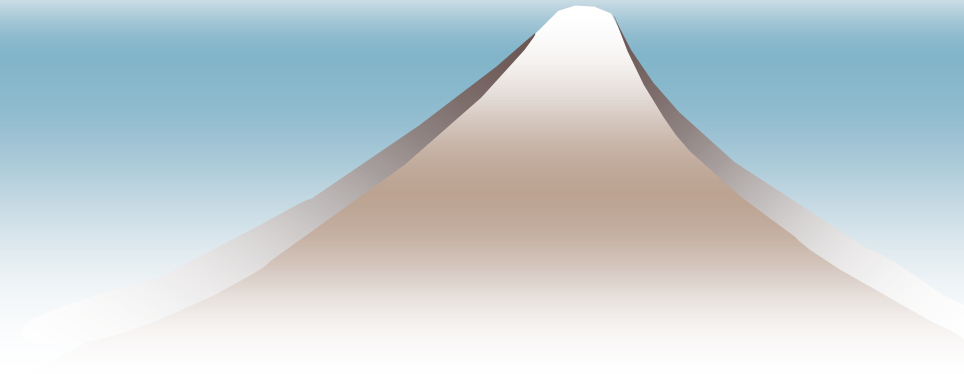
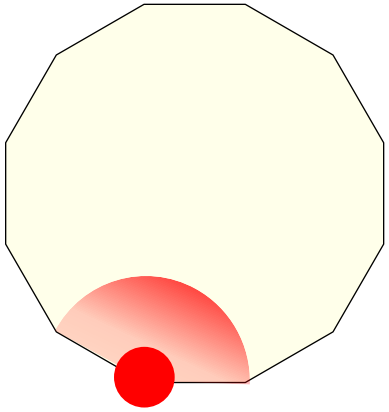
Kenta Noguchi (Tokyo Denki University)



Art Gallery Problem

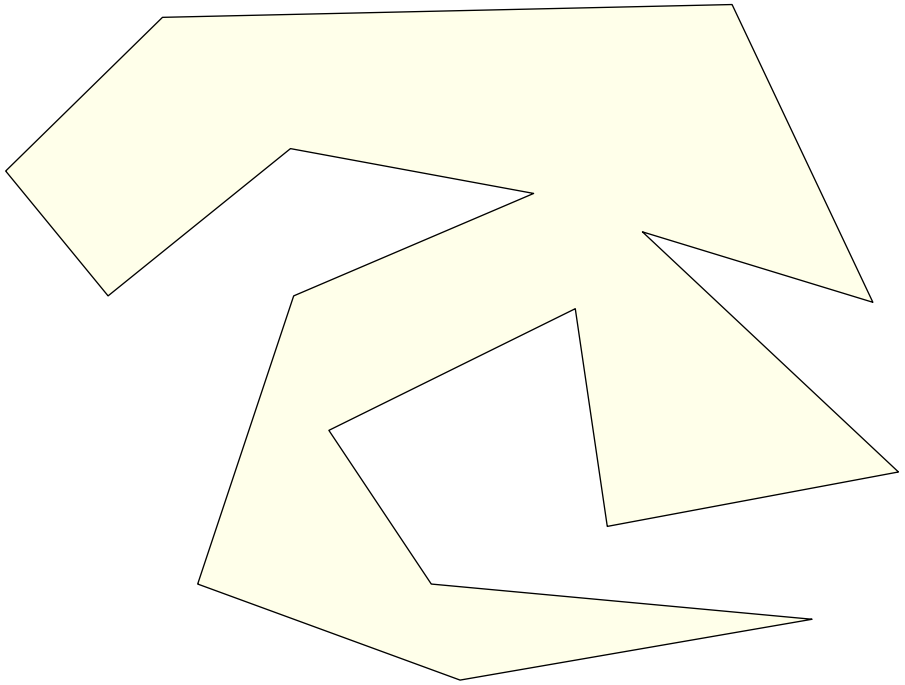
Art Gallery Problem :

How many guards do we need to watch everywhere in a museum?



One guard is enough

(non-convex) n-gon



$n/3$ guards are enough for any museum with n wall. (Chvatal, `75)

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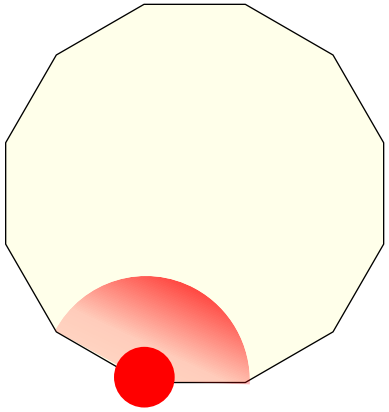

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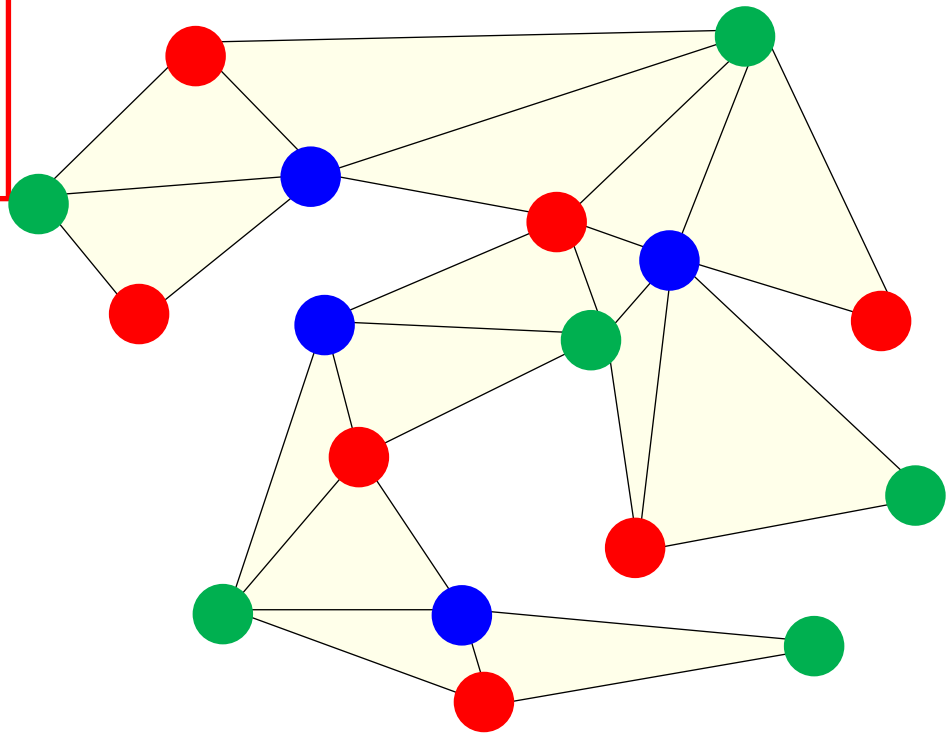

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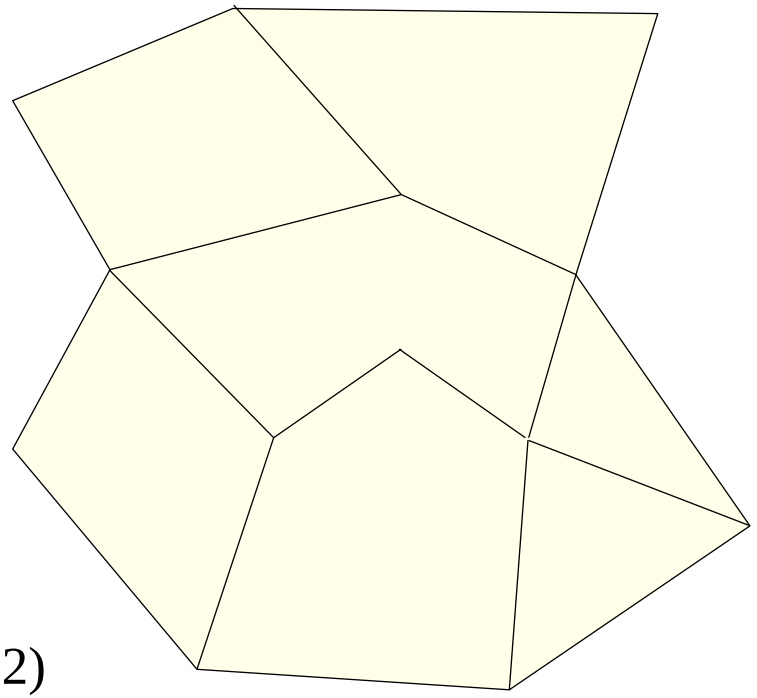
Key idea: Extension to 3-colorable triangulation

Prison Yard Problem

Prison Yard Problem :

How many guards do we need
to watch everywhere in a prison?

It has already rooms



c.f. Face Cover Problem

Find min. # of vertices hitting all faces

- NP-hard (Bienstock & Monma, `88)
- 27-approx. algorithm (Bienstock & Dean, 92)

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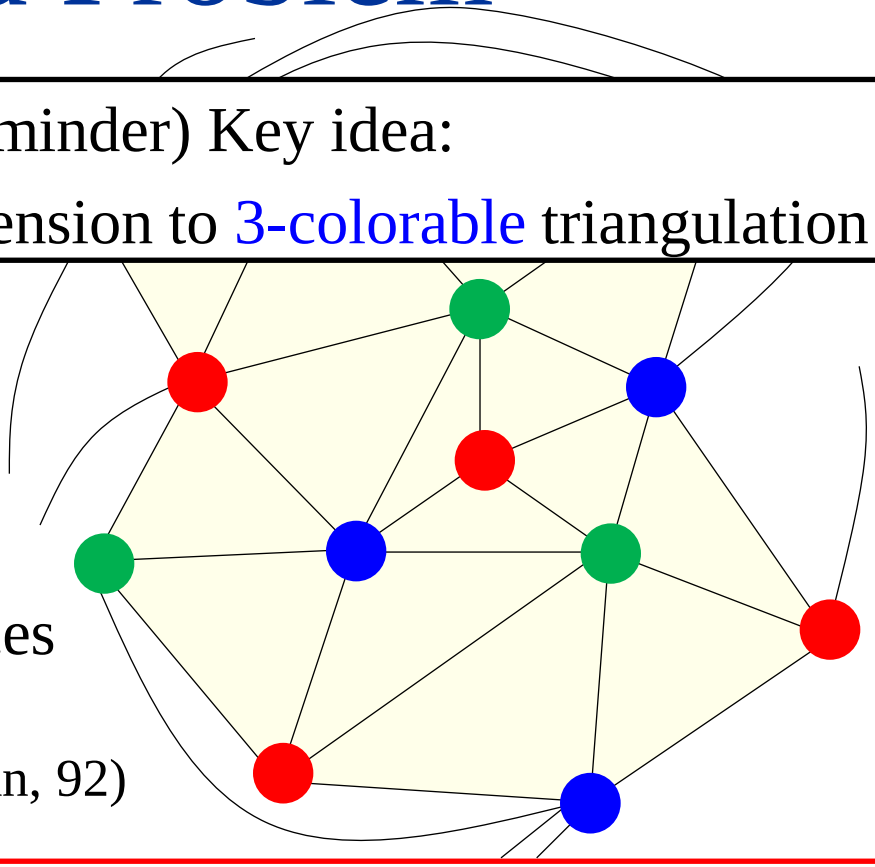
(Reminder) Key idea:

Extension to 3-colorable triangulation



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Find min. # of vertices hitting all faces

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Extension Problem :

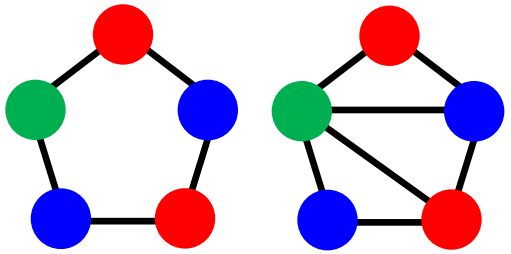
Determine if a graph can be extended to a 3-colorable tri.

Extension Problem

Extension Problem :

Determine if a graph on surface can be extended to a 3-colorable tri.

General plane graphs



NP-complete even for pentagulations

(Ext. prob. for pentagulations = 3-coloring prob.)

Quadrangulations (graphs in which all faces are quadlangular)

∀quad. has an extension to a 3-colorable tri.

$O(n^2)$ -algorithm (Hoffmann & Kriegel, `96)

$O(n)$ -algorithm (He & Zhang, `05) + const. delay enumeration

Extension Problem

Mosaics (graphs in which all faces are tri- or quadlangular)

Plane case :

A mosaic G —P→ 3-colorable tri = Eulerian tri.

const. delay enumeration
(Hoffmann-Ostenhof + α, `13+)

(all vertices have even deg.)

General surface case :

A mosaic G → 3-colorable tri

A mosaic G —P→ Eulerian tri.

Eulerian tri. —NP-c?→ 3-colorable tri

$$\begin{bmatrix} a_{11} & \cdots & a_{1g} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mg} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_g \end{bmatrix} = \begin{bmatrix} |C_1| \\ \vdots \\ |C_m| \end{bmatrix} \pmod 2$$

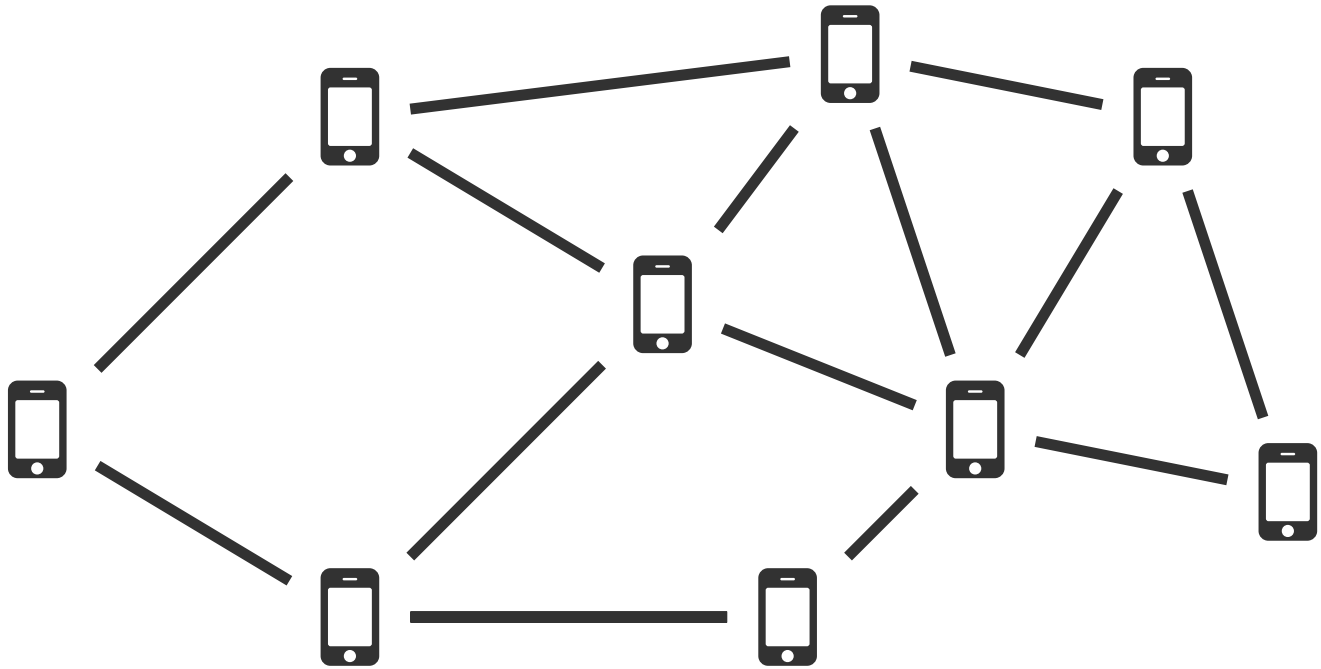
$$\begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & \ddots & \vdots \\ b_{g1} & \cdots & b_{gl} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_l \end{bmatrix} = \vec{0} \pmod 3$$

s.t. $y_j \in \{1, 2\}$ for $\forall j$

Constant-approximation algorithms for highly connected multi-dominating sets in unit disk graphs

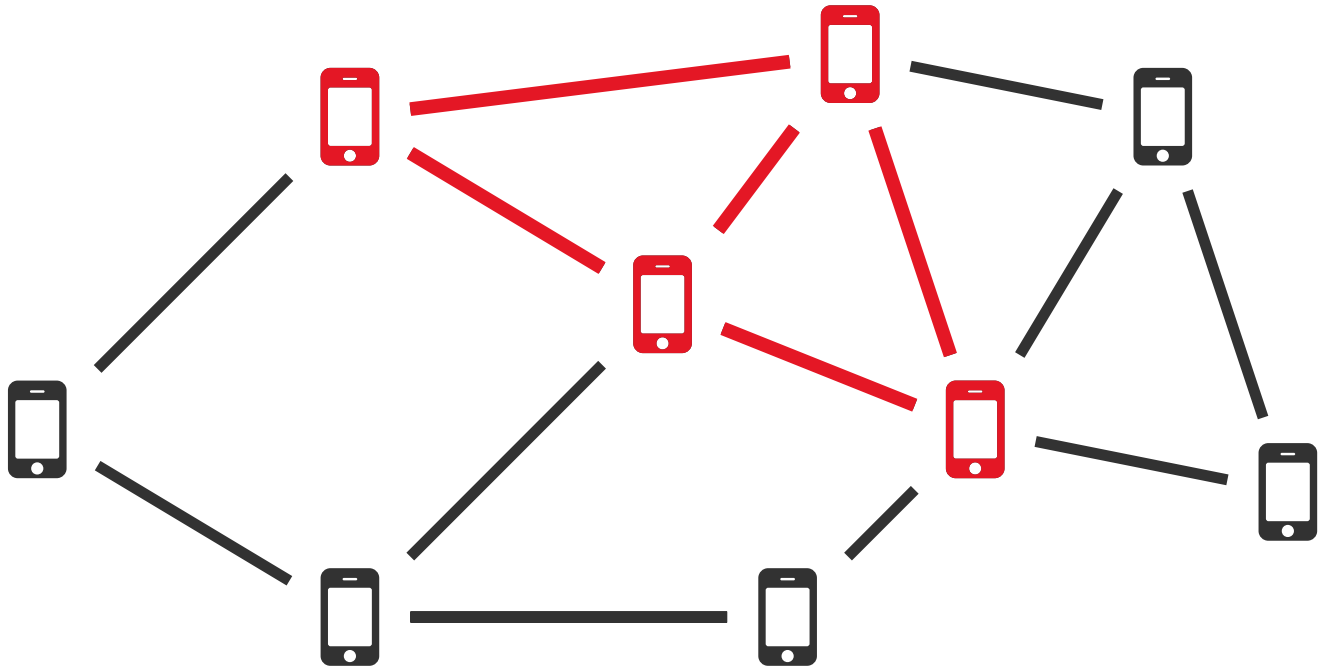
Takuro Fukunaga

Wireless ad hoc network



hard to keep global information

Wireless ad hoc network



hard to keep global information $\Rightarrow$ virtual backbone

k-connected m-dominating set in unit disk graphs

Find a small and stable virtual backbone
$\Rightarrow$ k-connected m-dominating set problem in a unit disk graph

Previous results

Constant-approximation algorithms are known only when

- $k \leq 3$ for uniform weights
- $k = 1, m = 1$ for general weights

Our results

First constant-approximation algorithms
for any (k, m) with a fixed constant $k \leq m$

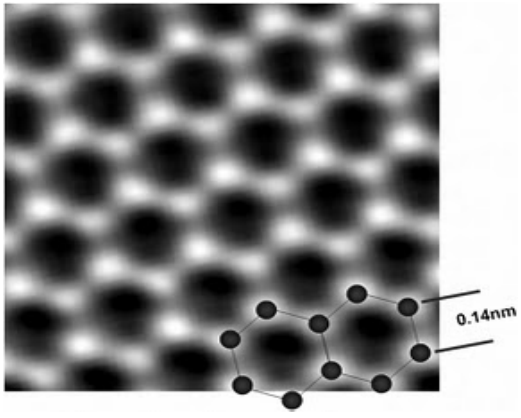
周期グラフ描画の回転対称性について

荒木徹也
(NII; Principles of Informatics Research Division Tokyo)

夫紀恵
(NII; JST, ERATO, Kawarabayashi Large Graph Project)

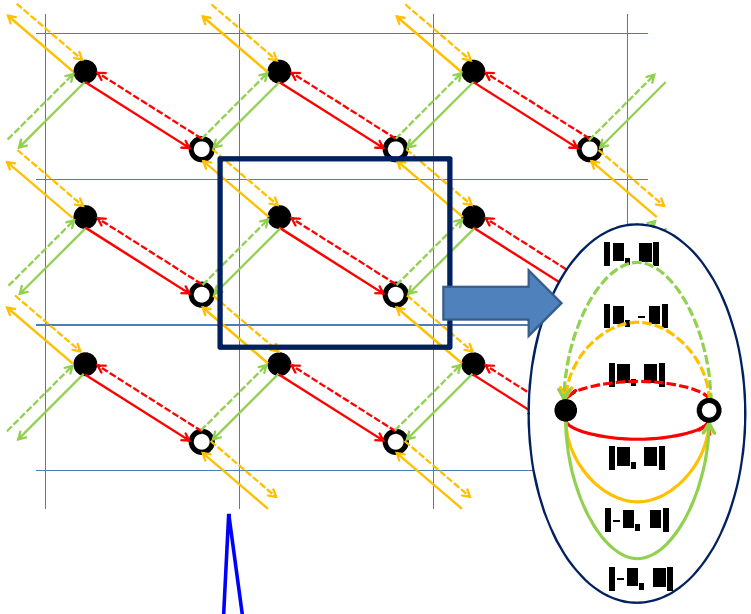
問題

周期 グラフ：結晶格子の組合せ構造を抽出
静的 グラフ ： 無限グラフである周期グラフを有限データで表現



TEM image of graphene sheet from Multifunctional polymer nanocomposites, CRC Press(2010)

6回転対称性を持つ描画



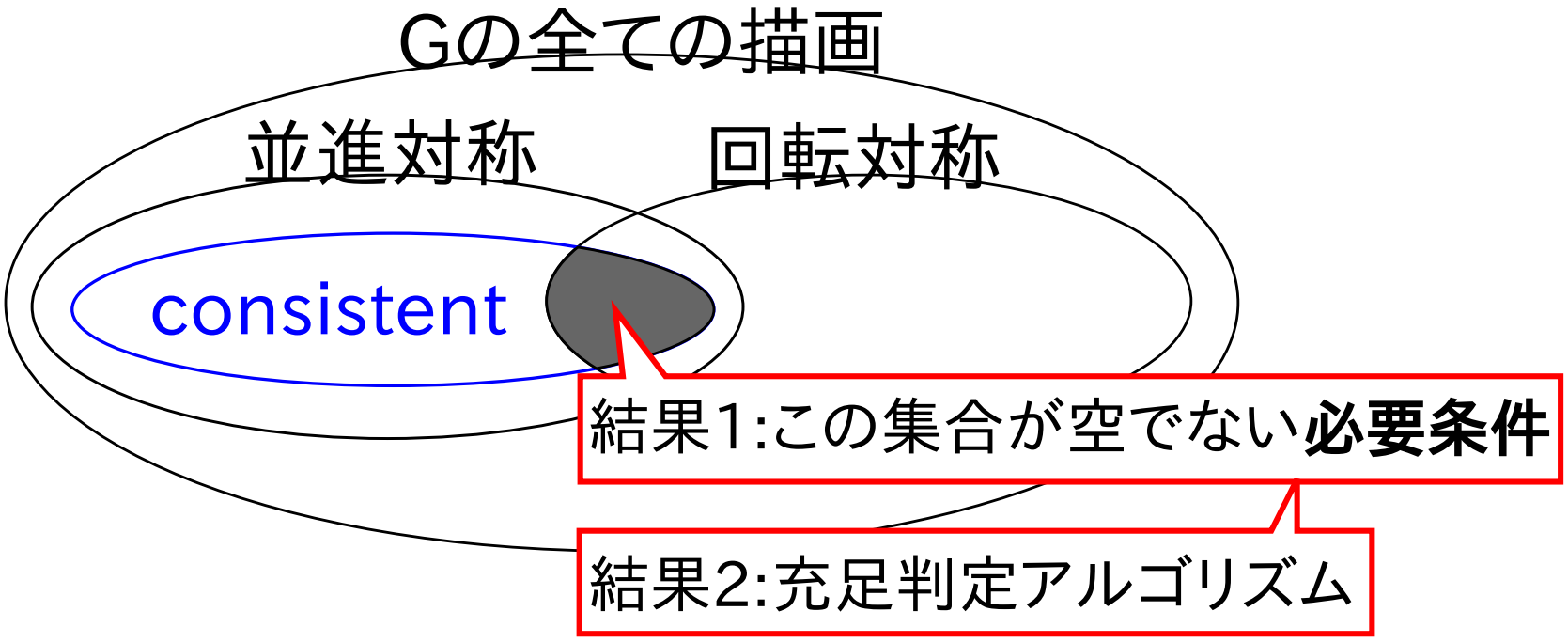
6回転対称性を持たない描画

疑問：静的グラフから，対応する周期グラフが並進対称性・回転対称性を持つ描画を持つかどうか判定できるか？

我々の結果

Gがコヒーレントであることを仮定する

多くの結晶が満たす性質



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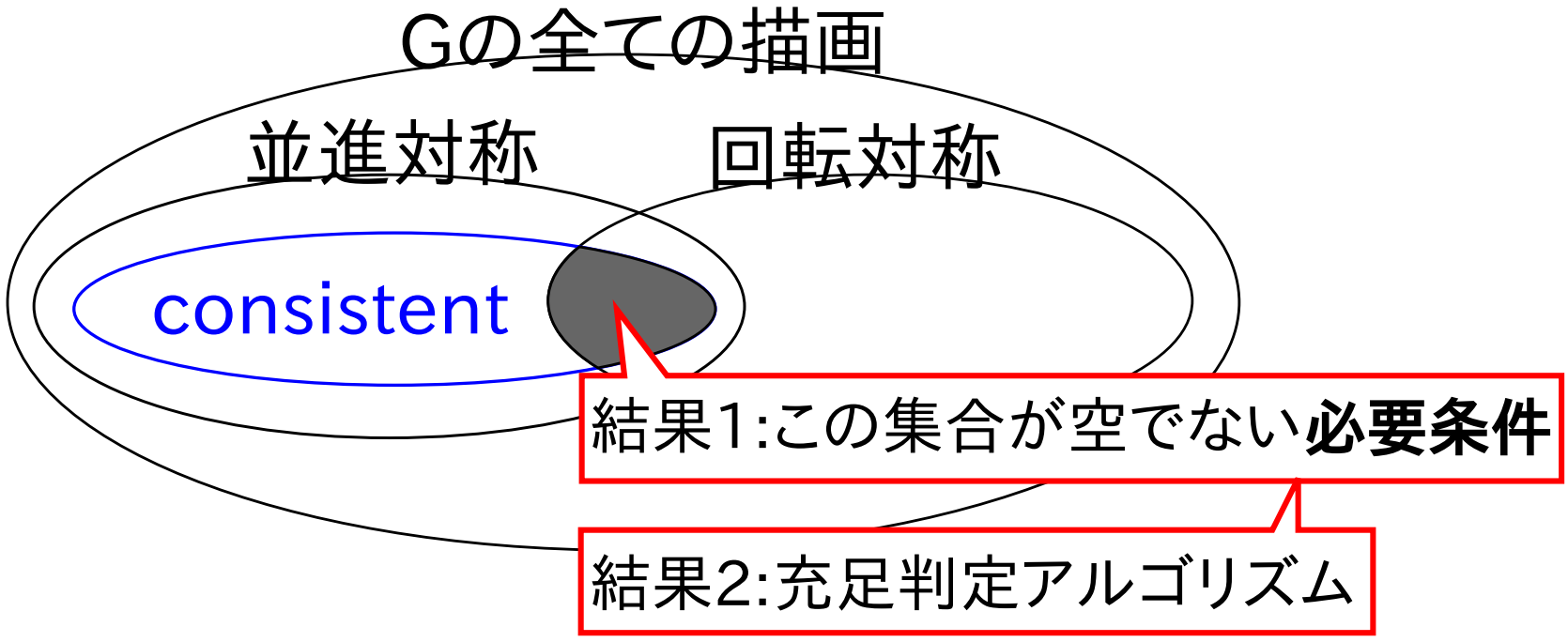

一部の周期グラフについて，回転対称性を持ち得ないことを証明するアルゴリズム