



# Stochastic Divergence Minimization for Online Collapsed Variational Bayes Zero Inference of Latent Dirichlet Allocation

@KDD2015



Issei Sato & Hiroshi Nakagawa

The University of Tokyo

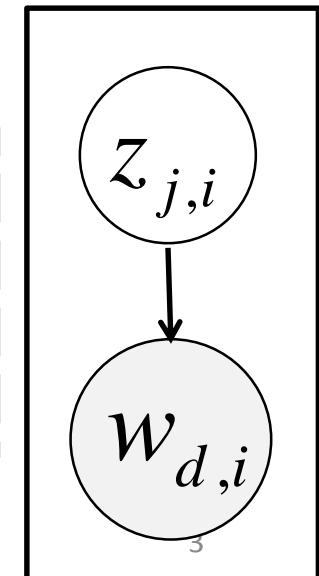
# Latent Dirichlet allocation [Blei,2003]

The annual ACM SIGKDD conference is the premier international forum for data mining researchers and practitioners from academia, industry, and government to share their ideas, research results and experiences.

# Latent Dirichlet allocation [Blei,2003]

The<sup>1</sup> annual<sup>2</sup> ACM<sup>2</sup> SIGKDD<sup>3</sup> conference<sup>2</sup> is<sup>1</sup> the<sup>1</sup> premier<sup>2</sup> international<sup>2</sup> forum<sup>2</sup> for<sup>1</sup> data<sup>3</sup> mining<sup>3</sup> researchers<sup>10</sup> and<sup>1</sup> practitioners<sup>10</sup> from<sup>1</sup> academia<sup>10</sup>, industry<sup>8</sup>, and<sup>1</sup> government<sup>7</sup> to<sup>1</sup> share<sup>6</sup> their<sup>1</sup> ideas<sup>10</sup>, research<sup>10</sup> results<sup>5</sup> and<sup>1</sup> experiences<sup>5</sup>.

Modeling co-occurrence:  
Frequently co-occurring words  
are assigned to the same topic (color)



# De Finetti theorem [De Finetti, 1930s]

A sequence of random variables  $(x_1, x_2, \dots)$  is infinitely exchangeable iff, for all  $n$

$$p(x_1, x_2, \dots, x_n) = \int \prod_{i=1}^n p(x_i | \theta) p(\theta) d\theta$$

# Latent Dirichlet allocation [Blei,2003]

Topic distribution for each document

$$\theta_d \sim \text{Dir}(\gamma) \quad (d = 1, \dots, D)$$

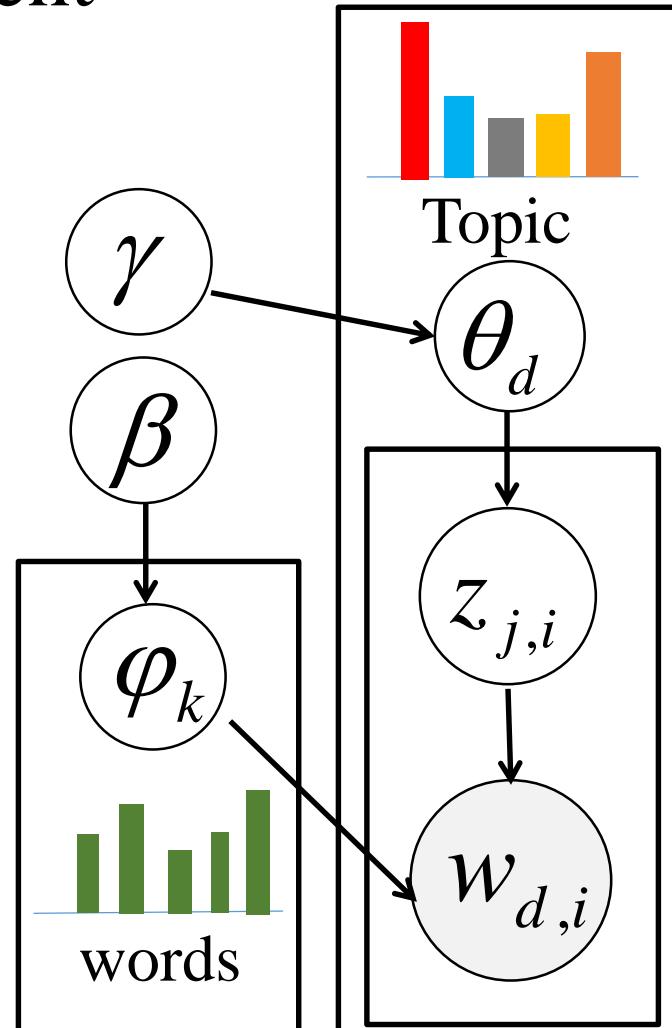
Word distribution for each topic

$$\phi_k \sim \text{Dir}(\beta) \quad (k = 1, \dots, K)$$

For each words:

$$z_{d,i} \sim \text{Multi}(\theta_d)$$

$$w_{d,i} \sim \text{Multi}(\phi_{z_{d,i}})$$



# Priors Matter [Wallach+, 2009]

## Asymmetric Dirichlet prior

$$\frac{\text{Dir}(\gamma_1, \gamma_2, \dots, \gamma_K)}{\text{Dir}(\beta_1, \beta_2, \dots, \beta_V)}$$

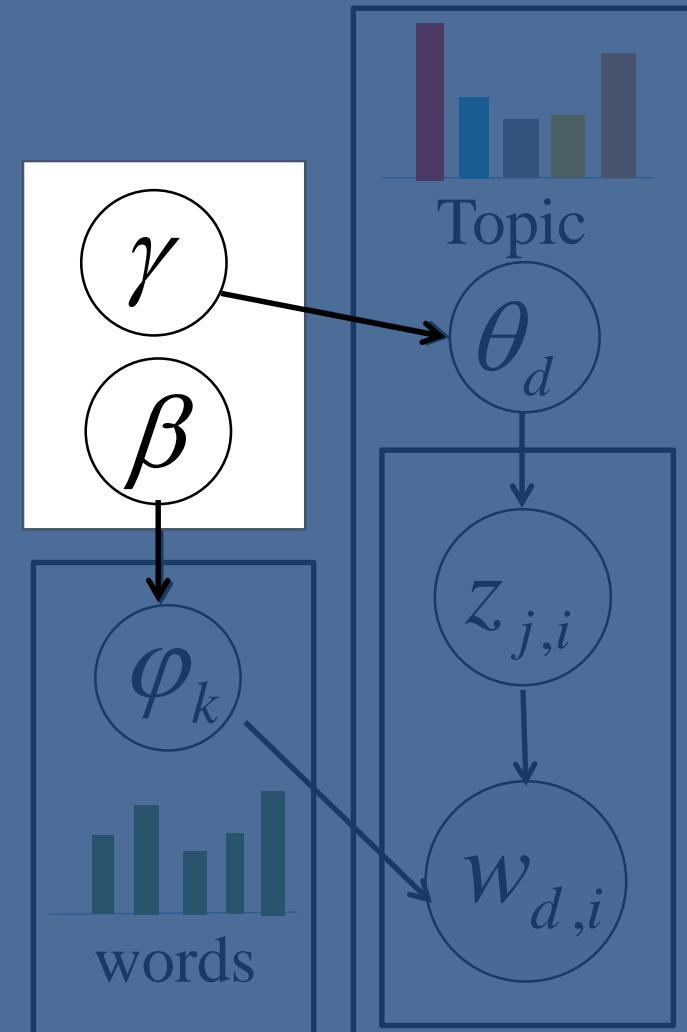
$$\frac{\text{Dir}(\gamma_1, \gamma_2, \dots, \gamma_K)}{\text{Dir}(\beta_1, \beta_2, \dots, \beta_V)}$$

## Symmetric Dirichlet prior

For each word:

$$\text{Dir}(\gamma, \gamma, \dots, \gamma)$$

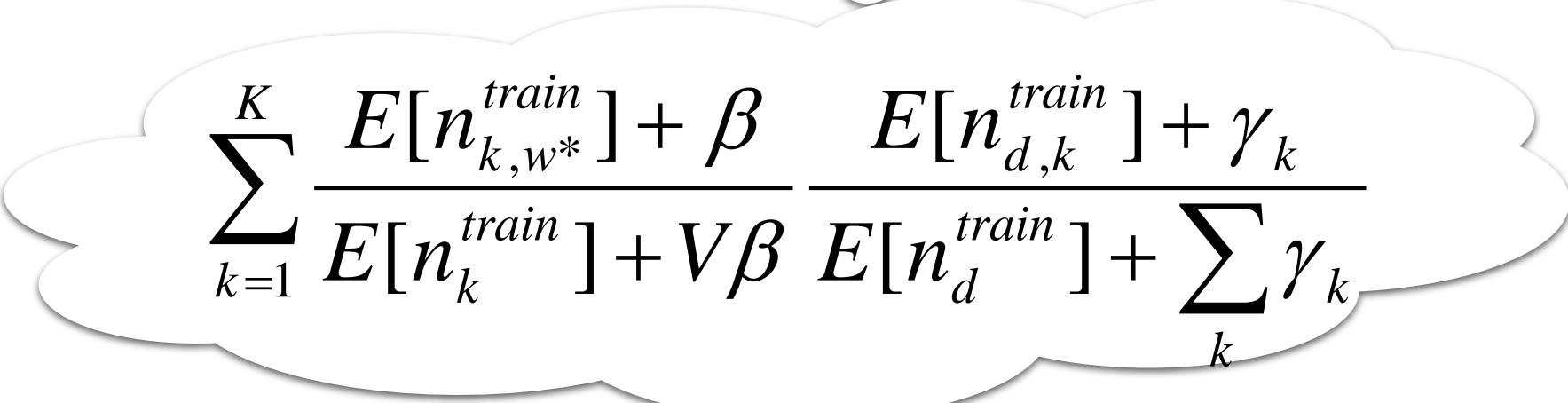
$$\text{Dir}(\beta, \beta, \dots, \beta)$$



# Evaluation: Perplexity

Prediction of held-out words

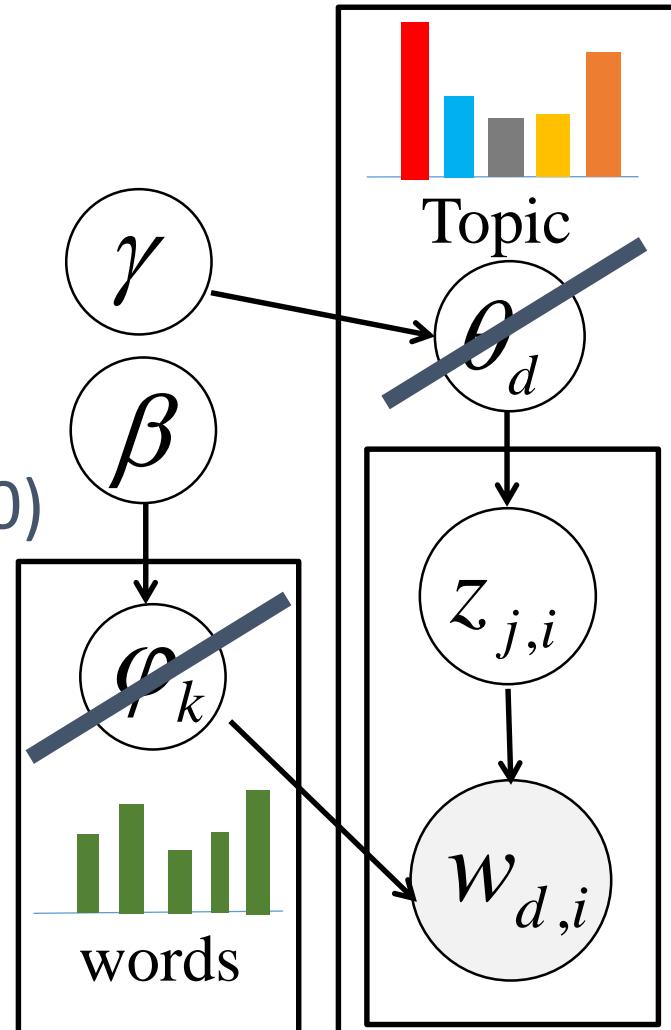
$$\exp \left[ \frac{1}{N_{test}} \sum_{w^* \in W_{test}} \log p(w^* | W_{train}) \right]$$


$$\sum_{k=1}^K \frac{E[n_{k,w^*}^{train}] + \beta}{E[n_k^{train}] + V\beta} \frac{E[n_{d,k}^{train}] + \gamma_k}{E[n_d^{train}] + \sum_k \gamma_k}$$

# Inference algorithms

- Variational Bayes (VB)  
[Blei+, JMLR2003]
  - Collapsed Gibbs Sampling (CGS)  
[Griffiths+, PNAS2004]
  - Collapsed Variational Bayes (CVB)  
[Teh+, NIPS2007]
  - Collapsed Variational Bayes Zero (CVZ)  
[Asuncion+, UAI2009]

# Marginalize out parameters



# Why CVB0 works better?

CVB0 uses zero-order Taylor approximation for  
expectations in CVB  
→ CVB0 is less accurate than CVB

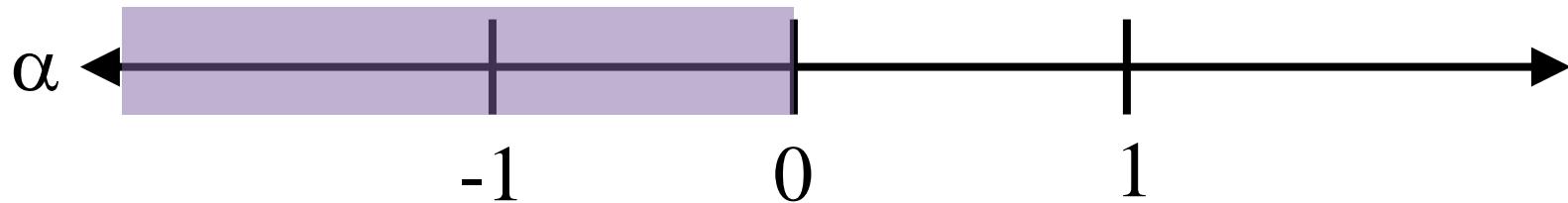
CVB0 can be formulated as  
a local  $\alpha$ -divergence minimization

[Sato & Nakagawa, ICML2012]

# $\alpha$ -divergence minimization

Inference	Marginalization	$\alpha$ -divergence
VB [Blei+,03]	—	$\alpha \rightarrow 0$
CVB [Teh+,07]	✓	$\alpha \rightarrow 0$
CVB0 [Asuncion+, 09]	✓	$\alpha \rightarrow 1 (\doteq 1)$
EP [Minka+,02]	—	$\alpha \rightarrow 1$

Zero forcing effect



The emphasis in the estimation is on high-frequency topics or low-frequency topics is forced to be zero

# Stochastic Optimization

Scaling up: Batch data → Sub-sampling

- Variational Bayes (VB) [Blei+, JMLR2003]

→ Stochastic Variational Bayes (SVB)

[Hoffman+, Sato+, NIPS2010]

- Collapsed Variational Bayes Zero (CVB0)

[Asuncion+, UAI2009]

→ Stochastic Collapsed Variational Bayes Zero (SCVB0)

[Foulds+, KDD2013]

# Framework of SCVB0

Problem

- How to formulate SO of CVB0
- CVB0 integrates out parameters

Solution

When we manually adjust Dirichlet prior,  
CVB0 update  $\sim$  MAP update.



SCVB0  $\sim$  Stochastic Approx. of MAP infer.

# Question and Problem on SCVB0

- Why MAP works better than VB ?
- We cannot use Asym. Dirichlet prior



## Our contribution

- Formulation of SCVB0  
→ Stochastic divergence minimization  
(SDM)
- Estimation of Dirichlet prior  
→ Reformulate DM of [Sato+, ICML2012]

# Main Idea

[Sato&Nakagawa, ICML2012]

Infer  $q(Z) = \prod_{d,i} q(z_{d,i})$  by DM  
=CVB0 update

This work



Infer

$q(Z, W | \gamma, \beta) = \prod_{d,i} q(z_{d,i} | w_{d,i}) \underline{q(w_{d,i} | \gamma, \beta)}$   
=CVB0 update ?  
by DM  
Stochastic Approx.

## Our Model

$$q(w_{d,i} | \gamma, \beta) = \sum_{k=1}^K \frac{E[n_{k,w_{d,i}}^{loo}] + \beta}{E[n_k^{loo}] + V\beta} \frac{E[n_{d,k}^{loo}] + \gamma_k}{E[n_d^{loo}] + \sum_k \gamma_k}$$

Leave-One-Out Perplexity

$$\exp \left[ -\frac{1}{N} \sum_{d,i} \log q(w_{d,i} | \gamma, \beta) \right]$$

Infer

$$q(Z, W | \gamma, \beta) = \prod_{d,i} q(z_{d,i} | w_{d,i}) q(w_{d,i} | \gamma, \beta)$$

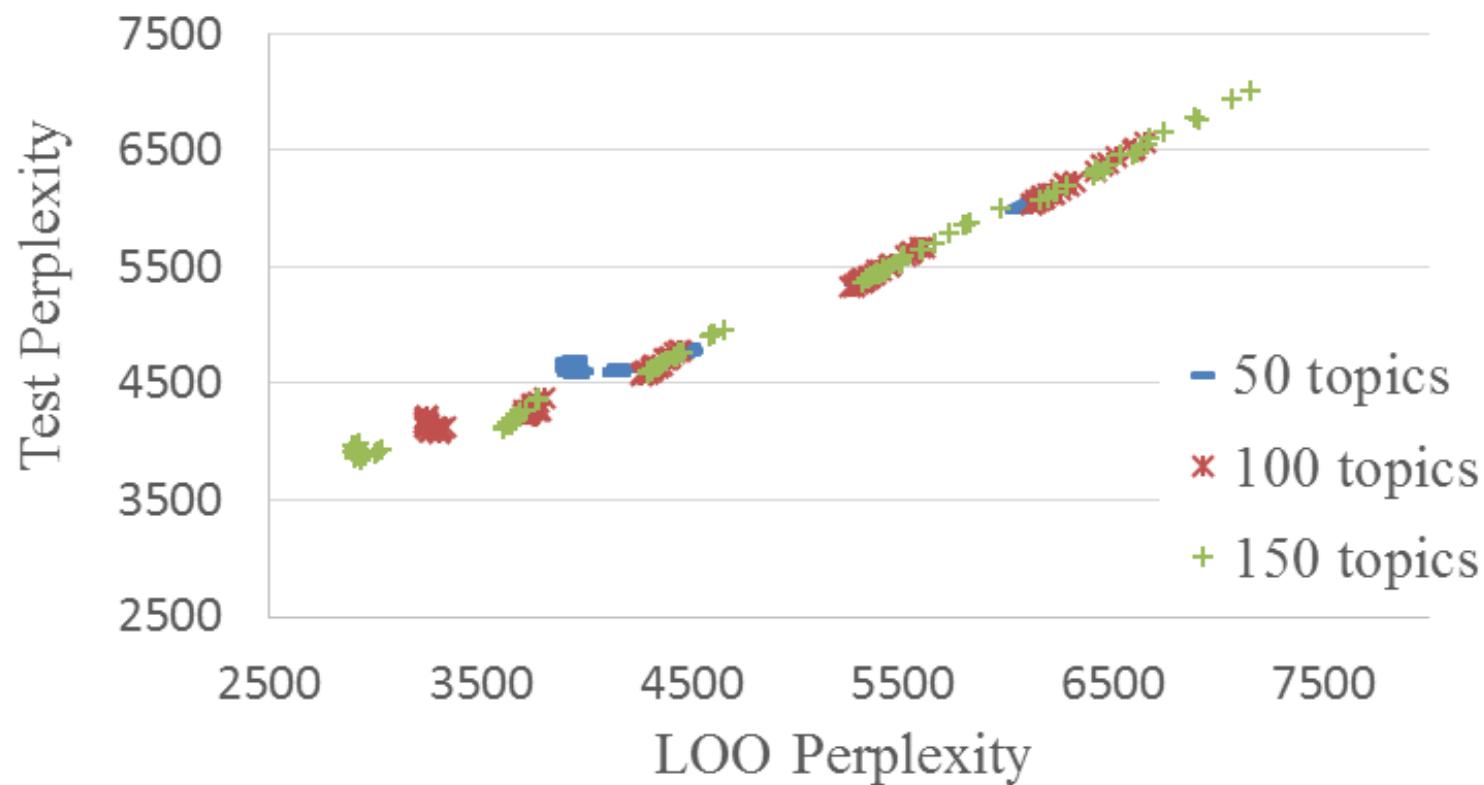
= CVB0 update ?

by DM  
Stochastic Approx.

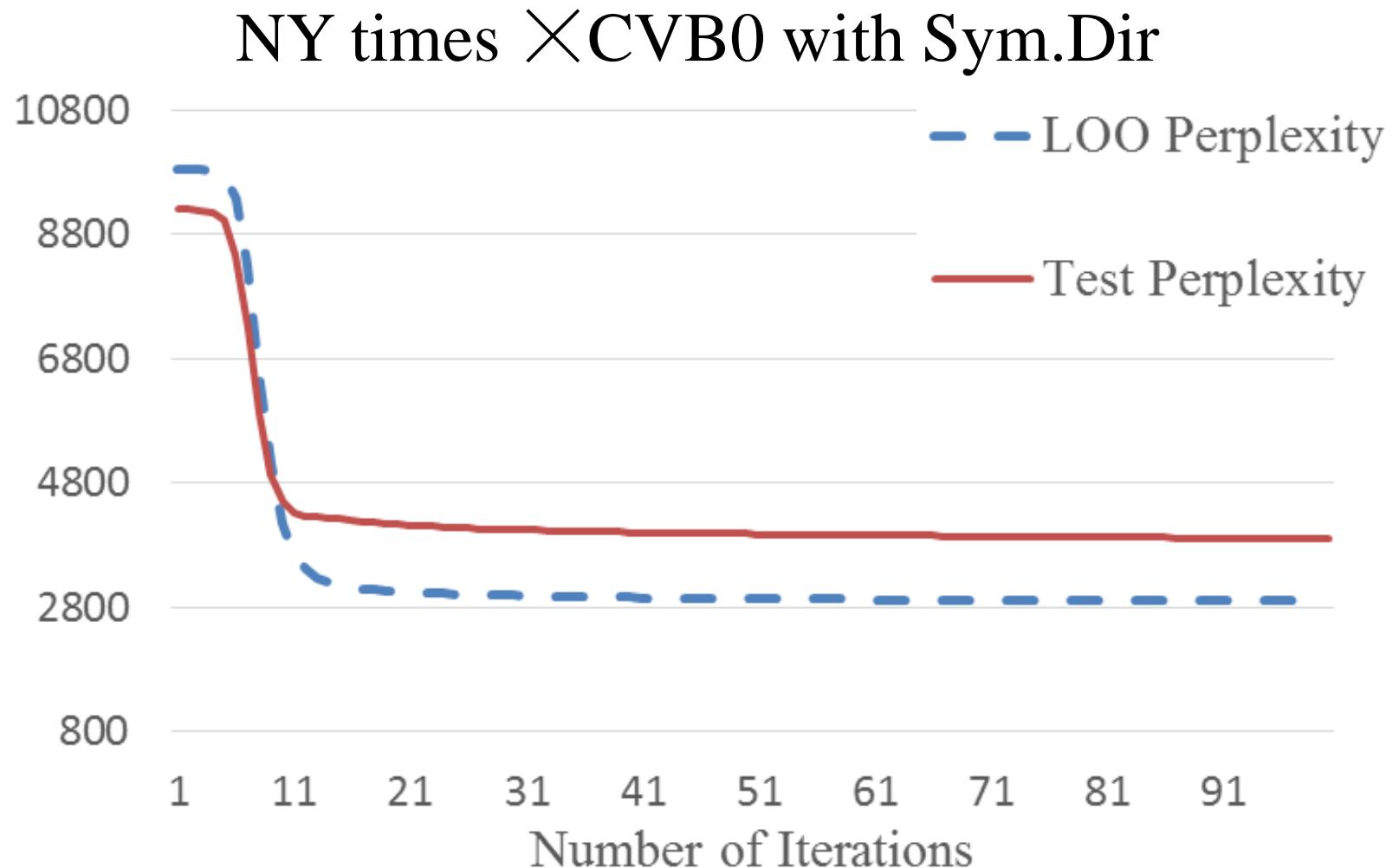
# Testset perplexity $\times$ LOO perplexity

NY times  $\times$  CVB0 with Sym.Dir

Correlation coefficient : 0.9913



# Testset perplexity × LOO perplexity



Empirical Bayes

$$(\gamma^*, \beta^*) = \arg \max_{\gamma, \beta} \log p(D | \gamma, \beta)$$

Variational EM

$$(\gamma^*, \beta^*) = \arg \max_{\gamma, \beta} L(D | \gamma, \beta)$$

$$\log p(D | \gamma, \beta) \geq L(D | \gamma, \beta)$$

This work

$$(\gamma^*, \beta^*) = \arg \max_{\gamma, \beta} \log q(D | \gamma, \beta)$$

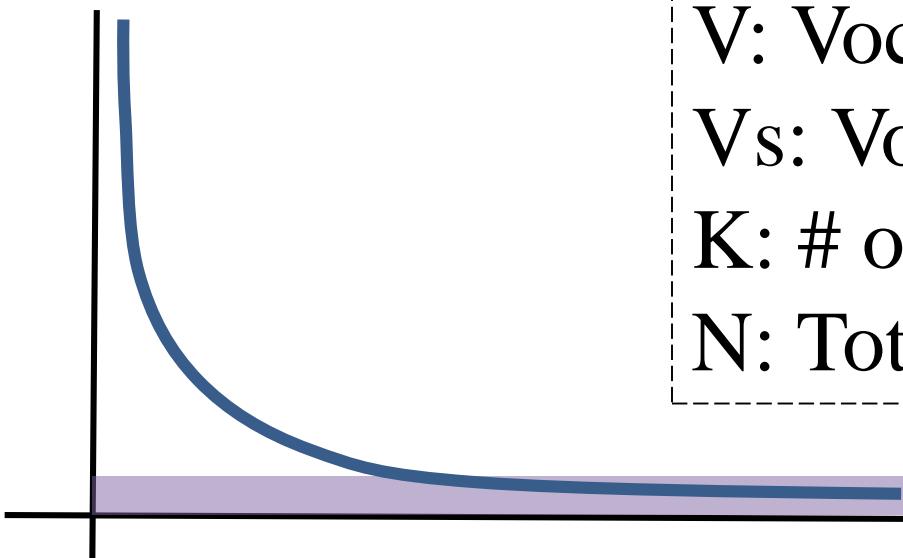
$\Leftrightarrow$  min Leave-One-Out Perplexity

$$\prod_{d,i} q(w_{d,i} | \gamma, \beta)$$

# Summary

	VB ‘03	SVB ‘10	CVB0 ‘07	SCVB0 ‘13	This work
Data processing	Batch	Sub- samp.	Batch	Sub- samp.	Sub- samp.
Memory	$O(VK)$	$O(VK)$	$O(NK)$	$O(VK)$	$O(VK)$
Update/mini-batch	-	$O(VK)$	-	$O(VK)$	$O(VsK)$
HDP(Asym. Dir)	✓	✓	✓	-	✓

V: Vocabulary size  
Vs: Vocab. size in sub-samples  
K: # of topics  
N: Total # of words



Ignore!

# Experimental settings

4 datasets

Dataset	# of Doc.	Vocab. size
DBLP	0.6M	19K
Wikipedia	1M	130K
Pubmed1M	1M	50K
Pubmed5M	5M	122K

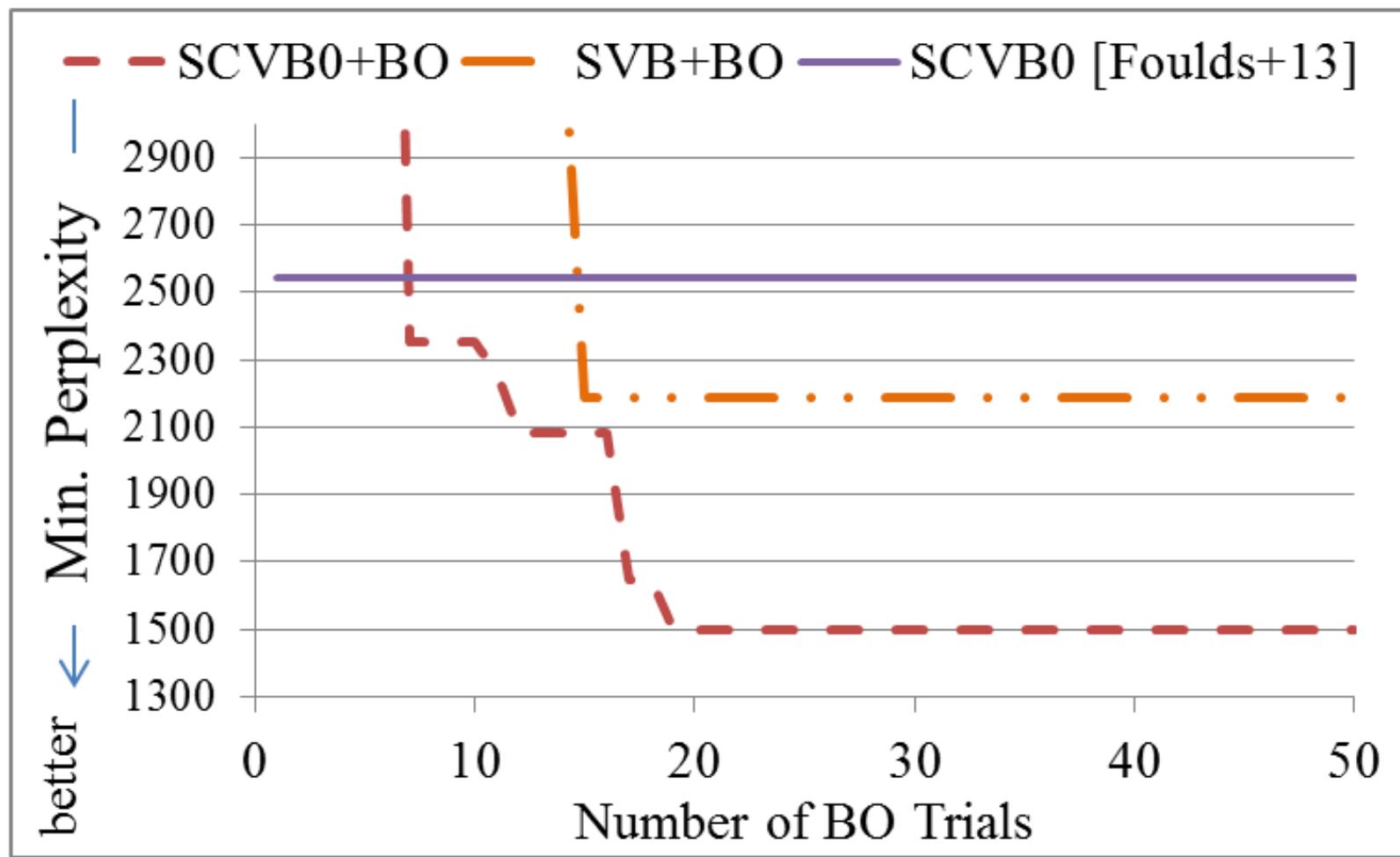
Evaluation: Testset Perplexity

Algorithms: SVB, SCVB0, SDM (This work)

# of Topics (Truncation level): 1000

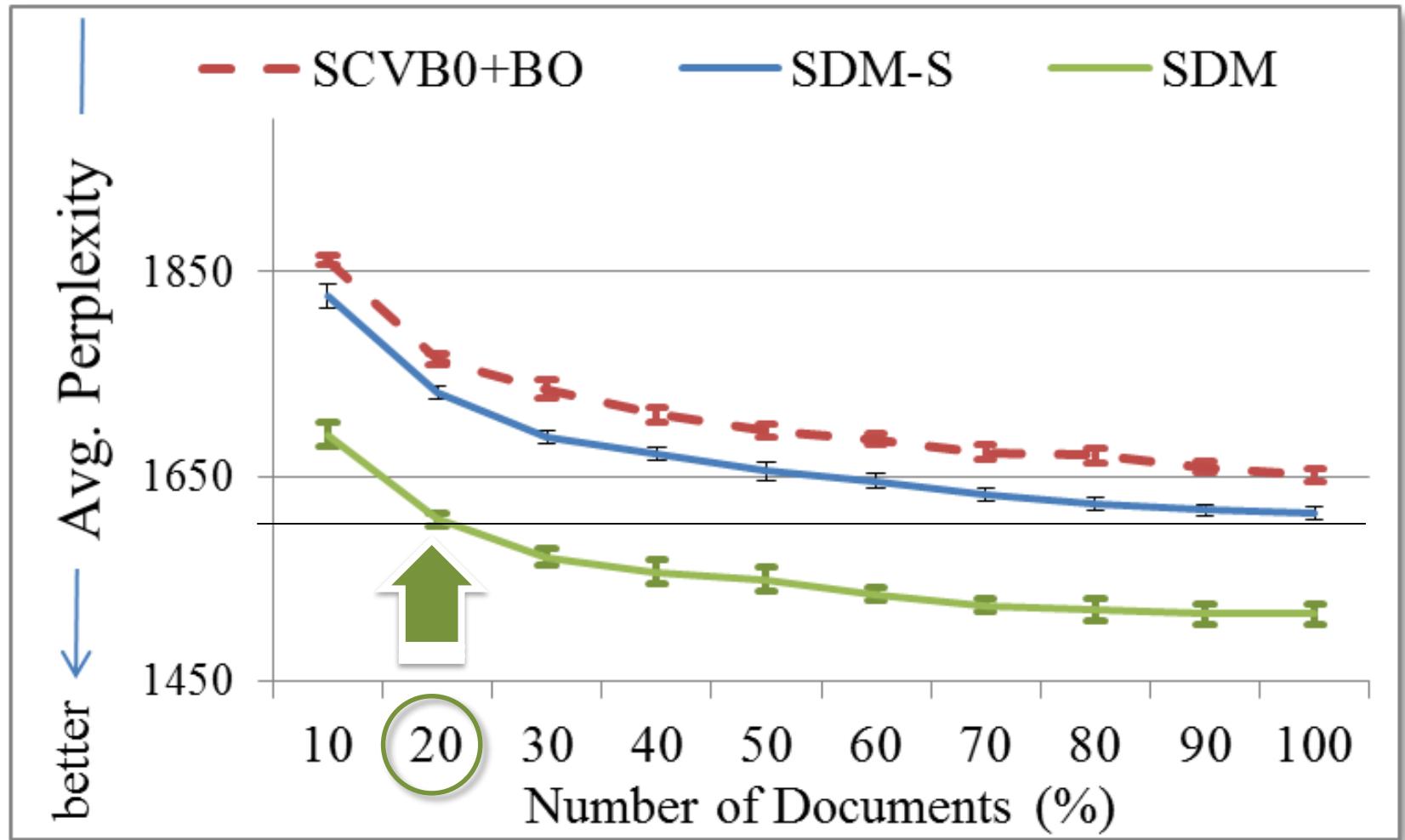
# Bayesian optimization for tuning hyper-parameters

Dataset: Pubmed1M



# Experimental result

Dataset: Pubmed5M



# 反省点：査読者との戦いを経て

- スコープが狭い
  - (建前) LDAは引用数1万を超える論文なので、LDAのアルゴリズムの改良自体は重要
  - (本音) 汎用性大事。少なくとも汎用性がありそうな書き方を心がけるべき
- 実験が少ない。Twitterの解析とかもやつたら?
  - (建前) [Foulds+, KDD2013]と同等の実験
  - (本音) 実験の種類はやはり多いほうが良い  
少なくとも[Sato+, KDD2012]のときは、Perplexity(4datasets), リンク予測(2datasets), 文書分類(2datasets)としたら褒められた
- 外部リンク先に証明のある定理は貢献に入れるべきではない
  - (建前)  
事前にCo-Chairsに可能か確認済なので、メタ査読者に確認を  
以前KDDの査読者に外部リンクに置くように言わされたことがある  
そのようにしているKDD論文も過去にある
  - (本音) Supplemental materialの無い会議では、やはりページ内に収めるように書くべき